Homework 1

Quantum Computation Theory
Instructor: Dr. Anna Vershynina

Deadline: February 4, 2020

1. Let Z be a phase-flip operator: $|+\rangle \to |-\rangle$ and $|-\rangle \to |+\rangle$. Find its matrix representation in a "+/-" basis and in a computation basis. Compare the results to the NOT operator.

2. Check the properties of the Pauli matrices: $Y = iXZ$, $X^2 = Y^2 = Z^2 = I$, $-iXYZ = I$, $[X,Y] = 2iZ$, $[Y,Z] = 2iX$, $[Z,X] = 2iY$.

3. Construct a two-qubit unitary that can copy two orthogonal states.

4. Prove that if a universal quantum cloner were to exist, then it would be possible for Alice to signal to Bob faster than the speed of light by exploiting only the entangled state $|\Phi^+\rangle^{AB}$ shared between them and no communication. That is, show the existence of a protocol that would allow for this. (Hint: One possibility is for Alice to measure the X or Z Pauli operator locally on her share of the entangled state, and then for Bob to exploit the universal quantum cloner.)

5. Find the transformation matrices between the following three bases: 1) Bell states, 2) computational basis, 3) $+/-$ basis in a two-qubit system.

6. Write a short summary of the most interesting news story that you've seen regarding quantum computation/information theory.