Homework 2

Quantum Computation Theory
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Deadline: March 5, 2020

1. Show that the evolution $\mathcal{N}(\rho) = \sum_m M_m \rho M_m^*$ is completely positive, where M_m are measurement operators such that $\sum_m M_m^* M_m = I$.

2. Verify that the action of the dephasing channel on the Bloch vector is
$$\frac{1}{2}(I + r_x X + r_y Y + r_z Z) \to \frac{1}{2}(I + (1-2p) r_x X + (1-2p) r_y Y + r_z Z) \ .$$

3. Let the density operator has the following matrix form
$$\frac{1}{2}\begin{bmatrix} 1+r_z & r_x - i r_y \\ r_x + i r_y & 1 - r_z \end{bmatrix} ,$$
where $\boldsymbol{r} = (r_x, r_y, r_z)$ is such that $\|\boldsymbol{r}\| \leq 1$. Show that the eigenvalues of a general qubit density operator with density matrix representation in the previous problem are as follows:
$$\frac{1}{2}(1 \pm \|\boldsymbol{r}\|).$$

4. Prove that a state $|\psi\rangle$ of a composite system AB is a product state if and only if it has Schmidt number 1. Prove that $|\psi\rangle$ is a product state if and only if ρ^A (and thus ρ^B) are pure states.

5. Show that randomly applying the Pauli operators I, X, Y , Z with uniform probability to any density operator gives the maximally mixed state
$$\frac{1}{4}\rho + \frac{1}{4}X\rho X + \frac{1}{4}Y\rho Y + \frac{1}{4}Z\rho Z = \frac{1}{2}I.$$

6. Show that the following operators are the Kraus operators for the quantum erasure channel
$$\sqrt{1-\epsilon}(|0\rangle^B \langle 0|^A + |1\rangle^B \langle 1|^A),$$
$$\sqrt{\epsilon}\,|\epsilon\rangle^B \langle 0|^A ,$$
$$\sqrt{\epsilon}\,|\epsilon\rangle^B \langle 1|^A .$$

7. Write a short summary of the most interesting news story that you've seen regarding quantum computation/information theory.