Honors Calculus, Math 1450: 17957- Assignment 2

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Due in class on 9/15/2017

1. Use implicit differentiation to find dy/dx

   (a) $y \sin\left(\frac{1}{y}\right) = 1 - xy$

   (b) $e^{2x} = \sin(x + 3y)$

   (c) $y^2 = e^{x^2} + 2x$

   (d) $2\sqrt{y} = y - x$

2. Find dy/dx

   (a) $y = \csc^{-1}(3x + 1)$

   (b) $y = e^{\cos x + \ln x}$

   (c) $\ln y = e^y \sin x$

   (d) $x^y = y^x$

3. Use logarithmic differentiation to find the derivative of y

   (a) $y = x^{\sin x}$

   (b) $y = (x + 1)^x$

   (c) $y = x(x + 1)(x + 2)$

   (d) $y = \sqrt{\frac{x}{x+1}}$

4. Show that $f'(x) \le f(x)$ if
$$f(x) = e^{\cos^2 x}$$

5. A particle is constrained to move along the real line so that its position as a function of time is equation is $x(t) = 2 \cos t + 4 \sin t$.

   (a) Find the velocity and acceleration as functions of time.

   (b) What is the velocity at time t $= 0$?

   (c) What is the first time $t > 0$ at which $x'(t) = 0$?

   (d) Show that the equation of motion satisfies the equation $x^2(t) + (x'(t))^2 = 20$.

6. If $y = A\sin(\ln x) + B\cos(\ln x)$, where A and B are constants, show that

$$x^2 y'' + xy' + y = 0.$$

7. When a circular plate of metal is heated in an oven, its radius increases at the rate of 0.01 cm/min. At what rate is the plates area increasing when the radius is 50 cm?

8. Two commercial airplanes are flying at an altitude of 40,000 ft along straight-line courses that intersect at right angles. Plane A is approaching the intersection point at a speed of 442 knots (nautical miles per hour; a nautical mile is 2000 yd). Plane B is approaching the intersection at 481 knots. At what rate is the distance between the planes changing when A is 5 nautical miles from the intersection point and B is 12 nautical miles from the intersection point?

9. Water is flowing at the rate of 50 m^3/min from a shallow concrete conical reservoir (vertex down) of base radius 45 m and height 6 m.

   (a) How fast (centimeters per minute) is the water level falling when the water is 5 m deep?

   (b) How fast is the radius of the waters surface changing then? Answer in centimeters per minute.

10. Use l'Hôpital's rule to find the limits

   (a) $\lim_{x \to 1} \frac{x-1}{\ln x - \sin \pi x}$
   (b) $\lim_{x \to \infty} x^2 e^{-x}$
   (c) $\lim_{x \to 0^+} (\ln x - \ln \sin x)$
   (d) $\lim_{x \to 0^+} (\csc x - \cot x + \cos x)$