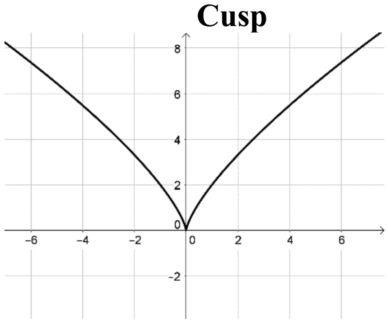
Lesson 13
Analyzing Other Types of Functions

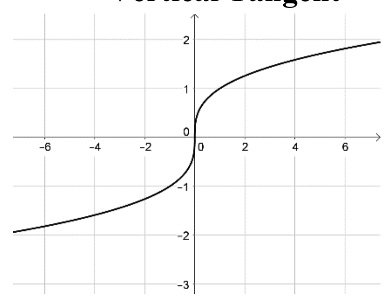
When working with functions different from polynomials, the critical numbers are defined as follows: The **critical numbers** of a function are numbers in the domain of the function where $f'(x)=0$ or where $f'(x)$ is undefined.

The derivative is undefined whenever a function has a cusp, vertical tangent, hole, vertical asymptote, or jump discontinuity. Examples are shown below.

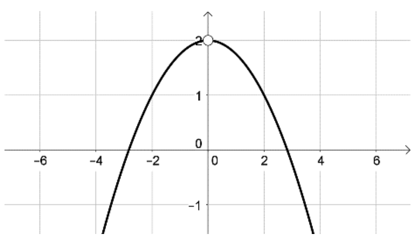
Cusp



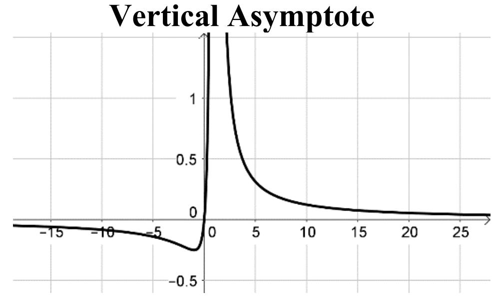
Vertical Tangent



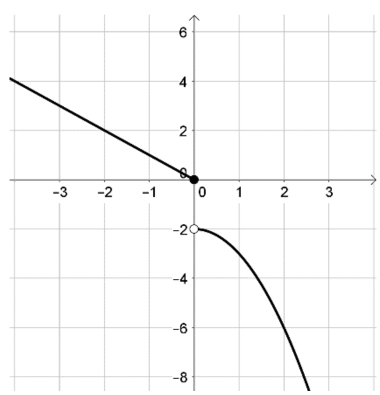
Hole



Vertical Asymptote



Jump Discontinuity



You must use caution when the graph of the derivative of a function is undefined since if this point is in the domain of the function, *it is a critical number.*

Example 1: The graph shown below is the graph of the derivative of $f(x) = \sqrt[3]{x} - x^2$. Give the domain of function *f* and its critical numbers.

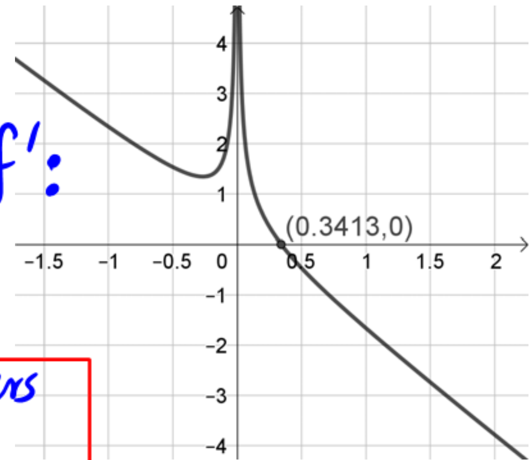
$y = \sqrt[3]{x} - x^2$

Domain: $(-\infty, \infty)$

Domain: $(-\infty, \infty)$

intersect:

f':



Domain of f:

$(-\infty, \infty)$

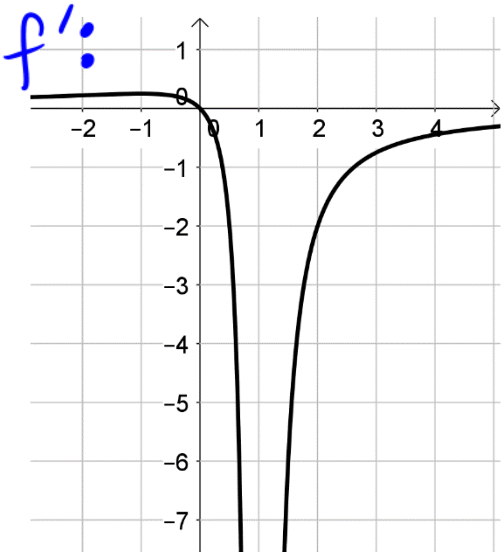
Critical numbers f:

$x = 0.3413$

$x = 0$

Example 2: The graph shown below is the graph of the derivative of a function *f*. The original function's domain is $(-\infty, 1) \cup (1, \infty)$. Find any critical numbers.

f':



C.N.

$x = 0$ only

not $x = 1$ b/c it is NOT in the domain of $f(x)$.

Next we'll analyze other types of functions almost the same way we analyzed polynomial functions in Lesson 12.

Recall the commands are slightly different:

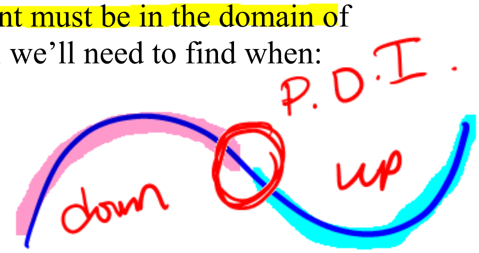
- Roots[<Function>, <Start x-Value>, <End x-Value>]
- Extremum[<Function>, <Start x-Value>, <End x-Value>]
- Asymptote[<function>]
- Inflectionpoint[<polynomial>] will NOT WORK FOR FUNCTIONS DIFFERENT FROM POLYNOMIALS. So we'll simply analyze the second derivative of a function to find any points of inflection.

A critical number is a number in the domain of the function f when $f'(x) = 0$ or when $f'(x)$ is undefined.

- Intervals where $f' > 0$, then f is increasing.
- Intervals where $f' < 0$, then f is decreasing.
- A relative maximum or a relative minimum can only occur at a **critical number**.
    - A function has a relative maximum if it changes from increasing to decreasing across a critical number.
    - A function has a relative minimum if it changes from decreasing to increasing across a critical number.

A point where concavity changes is a ***point of inflection***. This point must be in the domain of the function. In order to find the candidates of points of inflection, we'll need to find when:

- $f''(x) = 0$ or $f''(x)$ is undefined.
- Intervals where $f'' > 0$, then f is concave up.
- Intervals where $f'' < 0$, then f is concave down.



Example 3: Let $f(x) = 3(x-1)^{2/5} + 2x$. Find the function's domain, then find where the function is increasing/decreasing and any relative extrema. *Enter the function into GGB.*

$y = 3\sqrt[5]{(x-1)^2} + 2x$

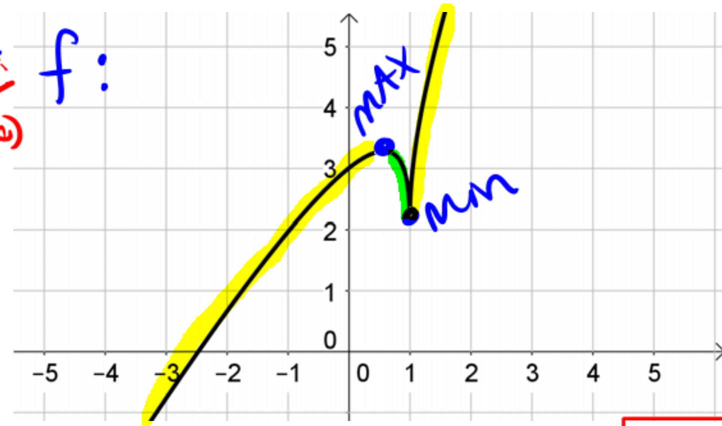
Domain: $(-\infty, \infty)$

Domain: $(-\infty, \infty)$

intersect

Domain of f: $(-\infty, \infty)$

f:



Commands:

extremum (f, −2, 4) → MAX (0.5732, 3.2805)

f(1) → 2   min (1,2)

~~Relative Maximum:~~

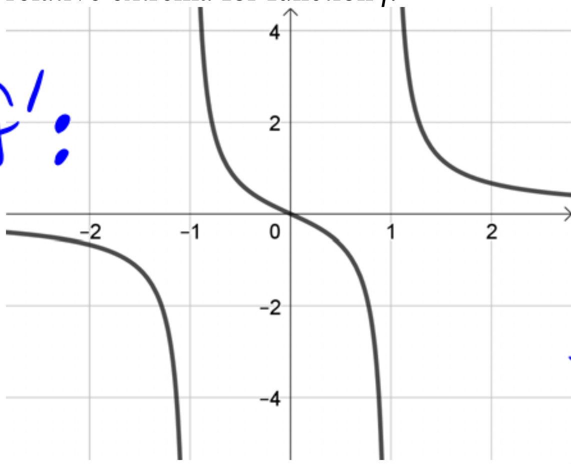
Relative Minimum:

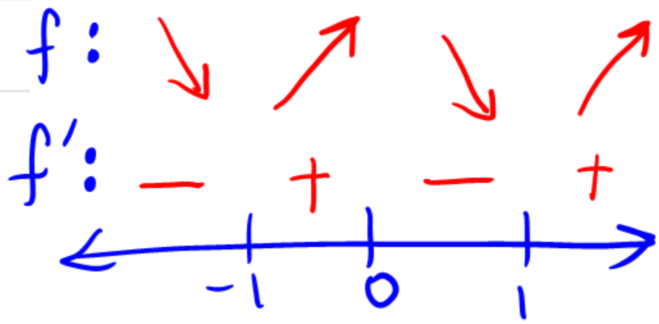
Increasing: $(-\infty, 0.5732) \cup (1, \infty)$

Decreasing: $(0.5732, 1)$

Example 4: The graph below is the graph of the first derivative of a function *f* whose domain is all real numbers except -1 and 1. Find any critical numbers, intervals of increase/decrease and relative extrema for function *f*.

f':





Critical Numbers:

$x = 0$ only

Relative Maximum:

at $x = 0$

Relative Minimum:

none

Increasing:

$(-1, 0) \cup [1, \infty)$

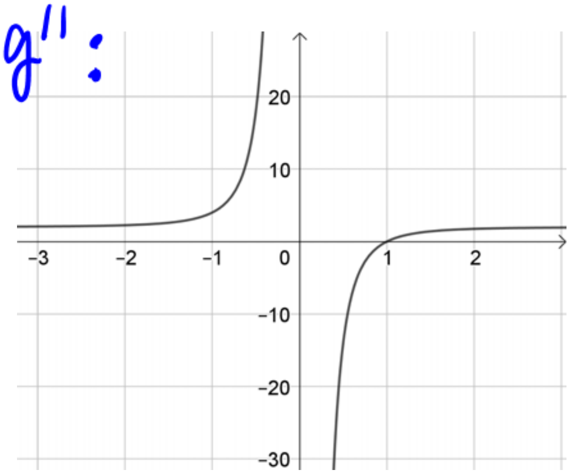
Decreasing:

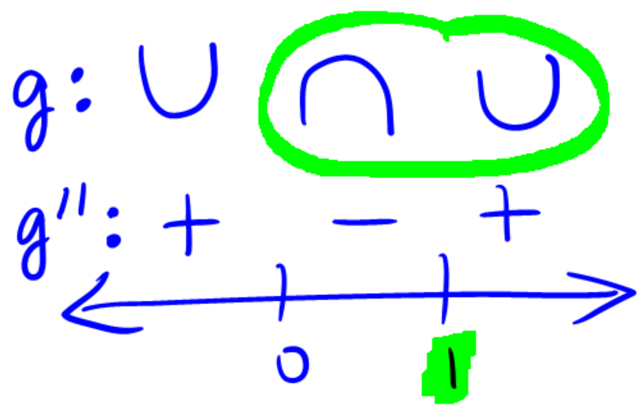
$(-\infty, -1) \cup (0, 1)$

Example 5: Let $g(x)=x^2-\frac{1}{x}$ find the function's domain, any intervals of concavity and any points of inflection. Domain: $(-\infty,0)\cup(0,\infty)$

Enter the function into GGB. The graph below is the graph of the second derivative of the function g.

g'':





~~Commands:~~

Concave Up: $(-\infty,0)\cup(1,\infty)$

Concave Down: $(0,1)$

Points of Inflection:

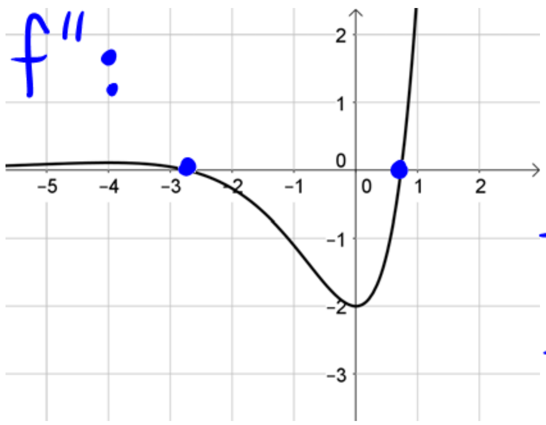
at $x=1$ and $y=g(1)=0$

$\boxed{\text{P.O.I. } (1,0)}$

Example 6: Let $f(x) = (x^2 - 2x)e^x + 1$. Find the function's domain, any intervals of concavity, and any points of inflection.

domain: $(-\infty, \infty)$

Enter the function into GGB. The graph below is the graph of the second derivative of the function f.



f: $\cup$ $\cap$ $\cup$

f'': + − +

−2.7 0.7

Commands:

roots(f'', −4, 2) $\longrightarrow$ (−2.7321, 0)
(0.7321, 0)

asymptote(f'') $\longrightarrow$ y = 0 H.A. not a V.A.

Concave Up:

$(-\infty, -2.7321) \cup (0.7321, \infty)$

Concave Down:

$(-2.7321, 0.7321)$

Points of Inflection:

at $x = -2.7321$, $f(-2.7321) = 1.8414$

at $x = 0.7321$, $f(0.7321) = -0.9302$