# Math 1431

## Section 16679

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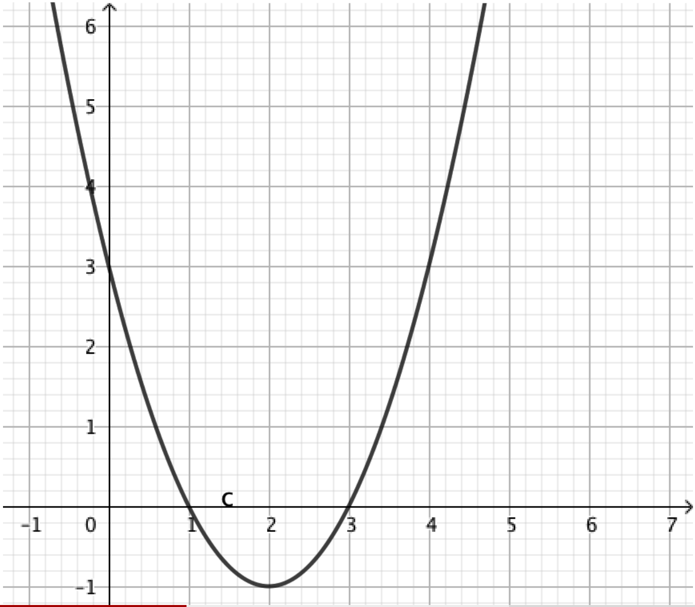
# Questions

# Quiz 1 Questions

# PT 1 Questions


6
5
4
3
2
1
-1
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-1
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1
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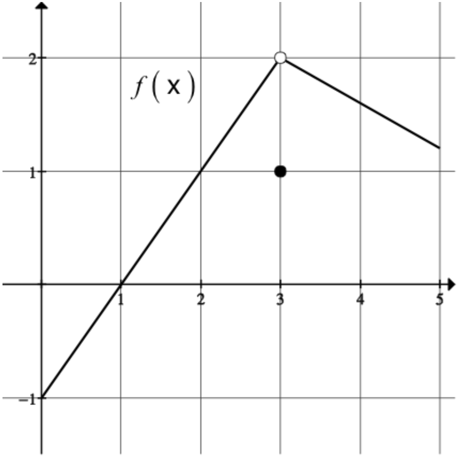
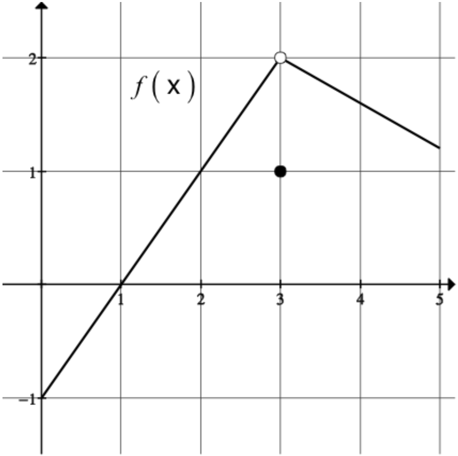
6
7

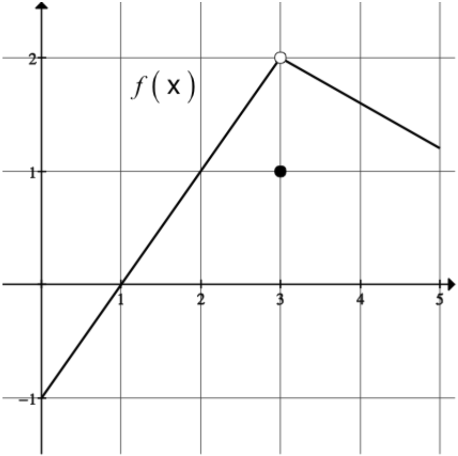
What is $f(2.999999)$ really close to?



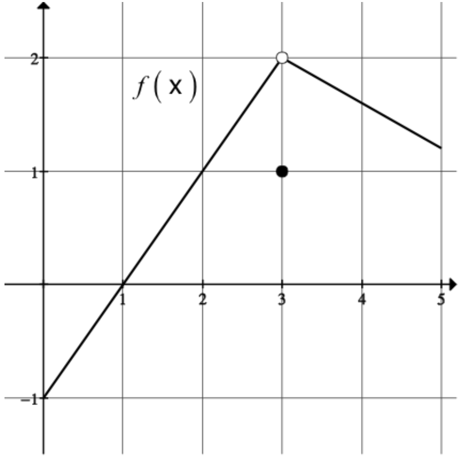
What is $f(3.0000001)$ really close to?



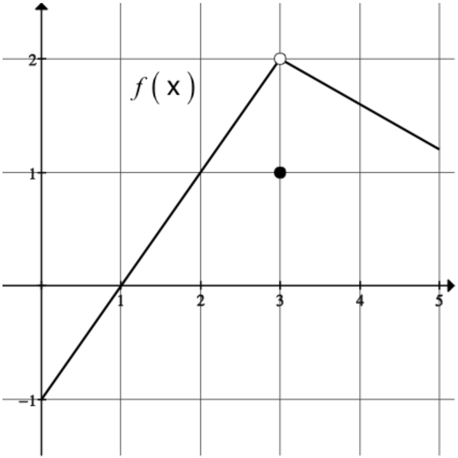
What do we "expect" $f(3)$ to be based on your answers to the previous two slides?

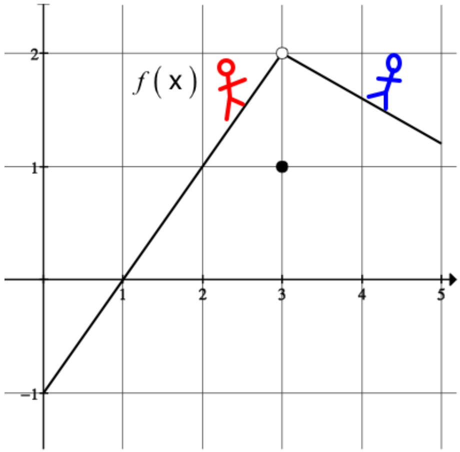


This expected value of $f(3)$ is the limit of this function as $x$ approaches 3.



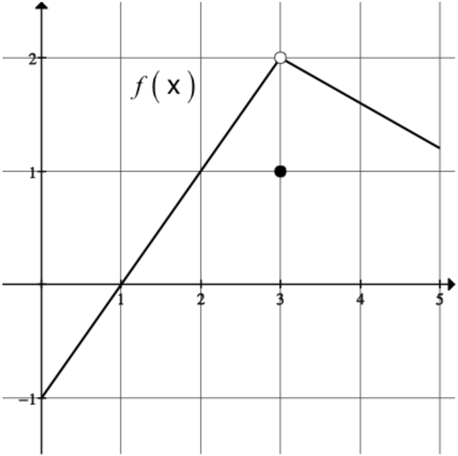
Notation: $\lim_{x \to 3} f(x) = 2$



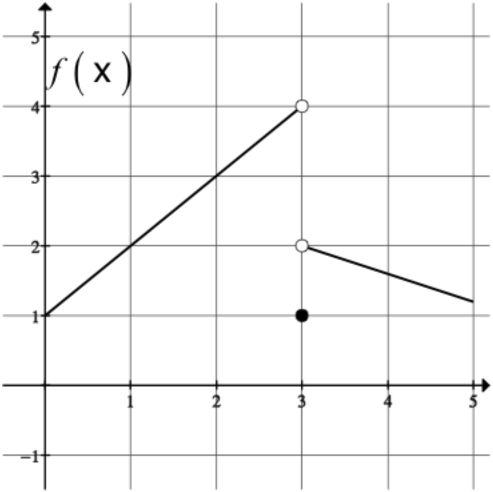


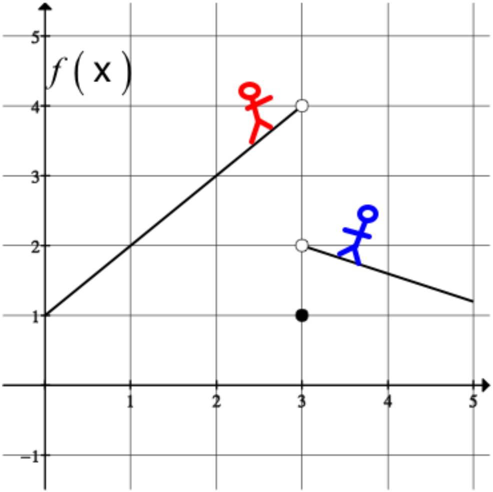
$f(x)$
2
1
−1
1
2
3
4
5

What is $f(3)$?



What is $\lim_{x\to 3} f(x)$?




f(x)
5
4
3
2
1
−1
1
2
3
4
5

Notation:

The limit as $x$ approaches $a$ from the left: $\lim\limits_{x \to a-} f(x)$

The limit as $x$ approaches $a$ from the right: $\lim\limits_{x \to a+} f(x)$

Rule:

For a limit to exist at $x = a$, we need $\lim\limits_{x \to a-} f(x) = \lim\limits_{x \to a+} f(x)$

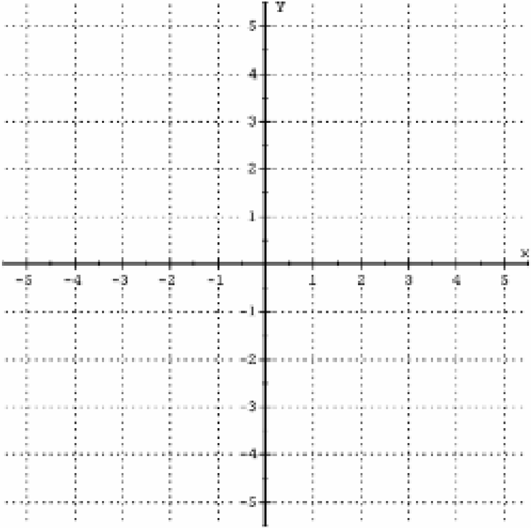
Below is the graph of $f(x) = \sin\left(\frac{1}{x}\right)$. What is $\lim_{x \to 0} f(x)$?

Let's look at $f(x) = \dfrac{|x|}{x}$.

To graph this, it helps to know what $|x|$ means:

So, $\dfrac{|x|}{x} =$

$f(x) = \dfrac{|x|}{x}$:



Given $f(x) = \dfrac{|x|}{x}$, find the following limits:

1. $\lim\limits_{x \to 2} f(x)$

2. $\lim\limits_{x \to -3} f(x)$

3. $\lim\limits_{x \to 0+} f(x)$

4. $\lim\limits_{x \to 0-} f(x)$

5. $\lim\limits_{x \to 0} f(x)$

**Limits: The Main Idea**

The limit of $f(x)$ as $x$ approaches the value "$a$" gives the behavior of $f(x)$ near $x = a$. To have a limit at $a$, a function must be defined everywhere in an open interval containing $a$ (except possibly at $a$ itself) and $L$ must be a number.

We say,

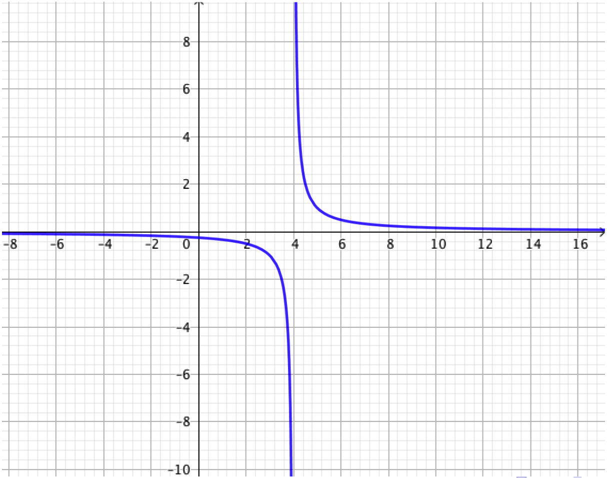
$$\lim_{x \to a} f(x) = L$$

if and only if

$$\lim_{x \to a-} f(x) = \lim_{x \to a+} f(x) = L$$

Below is the graph of $f(x) = \dfrac{1}{x-4}$.

What is $\lim_{x \to 4+} f(x)$? $\lim_{x \to 4-} f(x)$? $\lim_{x \to 4} f(x)$?



More examples:

1. $\lim\limits_{x \to 5} 2x + 1$

2. $\lim\limits_{x \to 3} \dfrac{x^2 - x - 6}{x - 3}$

3. $\lim\limits_{x \to 0} \sqrt{x - 6}$

4. $\lim\limits_{x \to 6+} \sqrt{x - 6}$

So, what conditions would make a limit not exist?

Express this limit in words and interpret its meaning:

$$\lim_{x\to 2} \frac{x^2 - 4}{x - 2} = 4$$

# Quiz 2 Examples

4. Evaluate the limit: $\lim_{x \to -4} \left( \dfrac{5}{x+4} \right)$

# Quiz 2 Examples

5. Evaluate the limit: $\lim_{x \to 0} \left( \frac{6x^2 - 7x}{x} \right)$

# Quiz 2 Examples

7. Evaluate the limit: $\lim_{x \to -4} f(x)$. Given that

$$f(x) = \begin{cases} 4x & x < -4 \\ -16 & x > -4 \end{cases}$$

# Quiz 2 Examples

10. Evaluate the limit: $\lim\limits_{x \to 2^+} f(x)$. Given that

$$f(x) = \begin{cases} 4x + 2 & x \leq 2 \\ x^2 - x & x > 2 \end{cases}$$

**Uniqueness of a limit**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} f(x) = M$ then $L = M$.

**The limit of a sum:**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c} (f(x) + g(x)) = L + M$ (provided each limit exists).

**The limit of a difference:**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c} (f(x) - g(x)) = L - M$ (provided each limit exists).

**The limit of a product:**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ then $\lim_{x \to c}(f(x)g(x)) = L \cdot M$ (provided each limit exists).

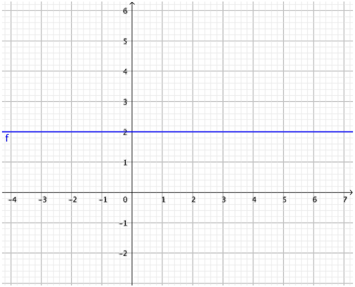
**The limit of a quotient:**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ with $M \neq 0$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$ (provided each limit exists).

Note:

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ with $L \neq 0$ and $M = 0$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)}$ does not exist.

Graph $f(x) = 2$

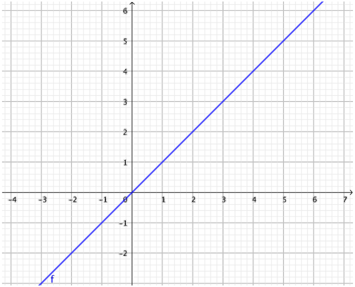



What is $\lim\limits_{x \to 4} f(x)$?
What is $\lim\limits_{x \to -1} f(x)$?
What is $\lim\limits_{x \to 0} f(x)$?

So, the limit of a constant function is that function:

If $f(x) = k$ then $\lim\limits_{x \to a} k = k$

Graph $f(x) = x$




What is $\lim_{x \to 4} f(x)$?
What is $\lim_{x \to -1} f(x)$?
What is $\lim_{x \to 0} f(x)$?

So: If $f(x) = x$ then $\lim_{x \to a} x = a$

What is a polynomial function?

A polynomial function is any function of the form:

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

Where $a_k$ is a real number and $n$ is an integer.

Examples:

$f(x) = x^2 - 4$

$g(x) = \frac{1}{3}x^5 + 2x^3 - x + 1$

$h(x) = 3x$

Because a polynomial function is just a combination of linear functions and constants and we know that we can find the limit for linear and constant functions, we can easily find the limit of polynomial functions.

So, to find $\lim_{x \to a} P(x)$, where $P(x)$ is a polynomial function, just plug in $a$. In other words,

$$\lim_{x \to a} P(x) = P(a)$$

What is a rational function?

A rational function, $R(x)$ is the quotient of two polynomial functions: $R(x) = \dfrac{P(x)}{Q(x)}$

Recall, **The limit of a quotient:**

If $\lim_{x \to c} f(x) = L$ and $\lim_{x \to c} g(x) = M$ with $M \neq 0$, then $\lim_{x \to c} \dfrac{f(x)}{g(x)} = \dfrac{L}{M}$

So, providing $\lim_{x \to c} Q(x) \neq 0$, we can just plug in to find the answer for a limit of a rational function.

However, if $\lim_{x \to c} Q(x) = 0$, we need to consider other things.

Let $P(x)$ and $Q(x)$ be polynomial functions and let $a$ be a real number. Then,

$$\lim_{x \to a} \frac{P(x)}{Q(x)} = \begin{cases} \frac{P(a)}{Q(a)} & \text{if } Q(a) \neq 0 \\ \text{undefined} & \text{if } P(a) \neq 0 \text{ and } Q(a) = 0 \end{cases}$$

If $P(a)$ and $Q(a)$ both equal 0 then more work is required.

Techniques to evaluate limits:

- direct substitution
- cancellation
- rationalization
- algebraic simplification

Examples:

1. $\displaystyle\lim_{x \to -2} (3x^2 + 1) =$

2. $\displaystyle\lim_{x \to 0} \frac{x^2 - 2x}{2x + 1} =$

3. $\displaystyle\lim_{x \to -1} \frac{2x^2 - x - 3}{x + 1} =$