Math 1431

Section 16679

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08/29/19

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You try: Give the largest δ that works with $\epsilon = 0.02$ for the limit

$$\lim_{x \to -1} (2x + 5) = 3$$

Questions

7. If $g(x) = \begin{cases} \frac{3x-6}{x-2} & x \neq 2 \\ 10 & x = 2 \end{cases}$, find $\lim_{x \to 2} g(x)$

8. $\lim_{x \to 0} \dfrac{\frac{1}{x+4} - \frac{1}{4}}{x} =$

Limits as $x \to \infty$

- $\lim_{x \to \infty} \frac{1}{x} =$

- $\lim_{x \to \infty} \frac{1}{x^2} =$

- $\lim_{x \to \infty} \frac{1}{x^n} =$

Examples:

1. $\lim_{x \to \infty} \frac{2x^2 - 3x + 1}{4x - x^2} =$

2. $\lim_{x \to -\infty} \dfrac{2x^2 - x + 5}{x^3 + x^2 + 1} =$

3. $\lim_{x \to \infty} \frac{3x^2 - 2x + 4}{\sqrt{x^4 + 3x^2 + 8}} =$

Section 1.3 - Definition of Limit and Arithmetic Rules

4. $\lim_{x \to \infty} \arctan(x) =$

5. $\lim_{x \to -\infty} \arctan(x) =$

Section 1.6 - The Pinching Theorem; Trig Limits

Suppose $f(x)$, $g(x)$ and $h(x)$ are defined on an open interval containing $x = c$ (except possibly at $x = c$).

If $f(x) \leq g(x) \leq h(x)$ and $\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$, then $\lim_{x\to c} g(x) = L$.

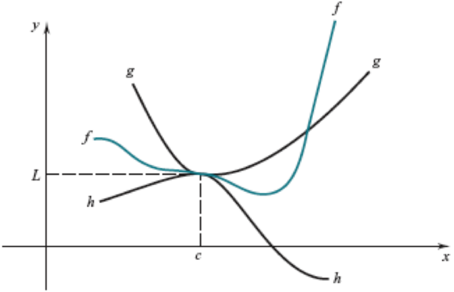


Figure 2.5.1

Section 1.6 - The Pinching Theorem; Trig Limits

Note: Trigonometric functions are continuous on their domain:

$$\lim_{x \to c} \sin(x) = \sin(c) \qquad \lim_{x \to c} \cos(x) = \cos(c)$$

Also, recall:

$$\sin(0) = 0 \text{ and } \cos(0) = 1$$

In the posted video, I use the Pinching Theorem to show:

$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Section 1.6 - The Pinching Theorem; Trig Limits

For any number $a \neq 0$, we have:

$$\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(ax)}{ax} = 0$$

Section 1.6 - The Pinching Theorem; Trig Limits

Examples:

1. $\lim_{x \to 0} \frac{\sin(5x)}{5x} =$

2. $\lim_{x \to 0} \frac{\sin(5x)}{x} =$

3. $\lim_{x \to 0} \frac{\sin(5x)}{2x} =$

For any number $a \neq 0$, we have:

$$\lim_{x \to 0} \frac{\sin(ax)}{ax} = 1 \quad \text{and} \quad \lim_{x \to 0} \frac{1 - \cos(ax)}{ax} = 0$$

More examples:

1. $\lim\limits_{x \to 0} \dfrac{x}{\sin(x)} =$

2. $\lim\limits_{x \to \pi/4} \dfrac{\sin(2x)}{x} =$

3. $\lim\limits_{x \to 0} \dfrac{1 - \cos(3x)}{x} =$

Continuity

A function f is said to be continuous at a point c if

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

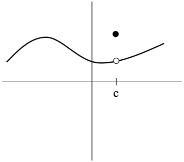
Can you give an example of a function where step 1 fails but step 2 doesn't fail?

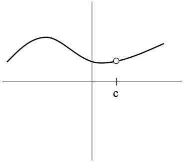
Can you give an example of a function where step 2 fails but step 1 doesn't fail?

Can you give an example of a function where step 3 fails but steps 1 and 2 don't fail?

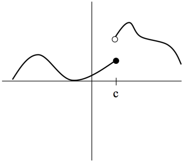
Types of discontinuity at a point

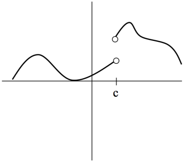
1. Removable:





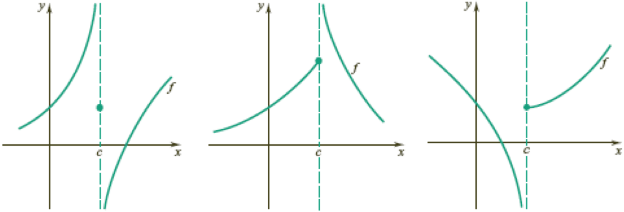
2. Non-Removable - Jump:





Types of discontinuity at a point

3. Non-Removable - Infinite:



If functions f and g are continuous at the point $x = c$, then

1. $f + g$ is continuous at c
2. $f - g$ is continuous at c
3. αf is continuous at c for each real number α
4. $f \cdot g$ is continuous at c
5. $\frac{f}{g}$ is continuous at c provided $g(c) \neq 0$

Lastly, - If g is continuous at c and f is continuous at $g(c)$, then the composition $f \circ g$ is continuous at c.

Section 1.4 - Continuity

Where are polynomials continuous?

Where are rational functions continuous?

There is also **One-Sided Continuity**

A function is continuous from the left at c if $\lim_{x \to c-} f(x) = f(c)$

and

it is continuous from the right at c if $\lim_{x \to c+} f(x) = f(c)$

Examples: Discuss the continuity for each function.

1. $f(x) = \dfrac{x+2}{x^2 - x - 6}$

2. $f(x) = \dfrac{x^2 + 2x}{x^2 - 4}$

3. $f(x) = \dfrac{x+5}{x^2+5}$

4. $f(x) = \dfrac{x+5}{x^2+5x}$

5. $f(x) = \sqrt{x-3}$

6. $f(x) = \dfrac{\sqrt{x} - 1}{x^2 + 4x - 5}$

7. $f(x) = \begin{cases} x^3 & x < 1 \\ \sqrt{x} & x \geq 1 \end{cases}$

8. $f(x) = \begin{cases} 6 & x \leq -2 \\ -6 & x > -2 \end{cases}$

9. $g(x) = \begin{cases} x+2 & x < -2 \\ \sqrt{4-x^2} & -2 \le x < 2 \\ 1 & x = 2 \\ x-2 & x > 2 \end{cases}$

10. Find c so that $h(x)$ is continuous. $h(x) = \begin{cases} 2x - 3 & x < 2 \\ cx - x^2 & x \geq 2 \end{cases}$

Some more

1. Determine if the following function is continuous at the point where x = 3.

$$g(x) = \begin{cases} 2x^2 + 9 & x < 3 \\ 27 & x = 3 \\ x^3 & x > 3 \end{cases}$$

2. Discuss the continuity of $f(x) = \begin{cases} -x^2 & x < -1 \\ 3 & x = -1 \\ 2 - x & -1 < x \leq 1 \\ \frac{1}{x^2} & x > 1 \end{cases}$

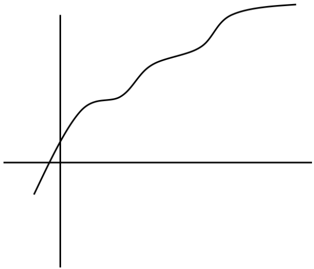
3. Find A and B so that $f(x)$ is continuous.

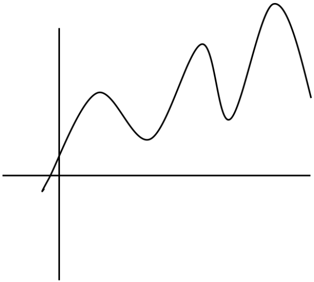
$$f(x) = \begin{cases} 2x^2 - 1 & x < -2 \\ A & x = -2 \\ Bx - 3 & x > -2 \end{cases}$$

Section 1.5 - The Intermediate Value Theorem

A very important result of continuity is the **Intermediate Value Theorem**.

If $f(x)$ is continuous on the closed interval $[a, b]$ and K is a value between $f(a)$ and $f(b)$, then there is at least one value c in (a, b) such that $f(c) = K$.





Examples:

Use the intermediate value theorem to show that there is a solution to the given equation in the indicated interval.

1. $x^2 - 4x + 3 = 0$ on the interval $[2, 4]$

2. $x^3 - 6x^2 - x + 2 = 0$ on the interval $[0, 3]$

3. $2\tan(x) - x = 1$ on the interval $\left[0, \frac{\pi}{4}\right]$

4. Show there is a value of x between 1 and 3 so that $-3x^3 + 2x^4 = 7$

5. Does the Intermediate Value Theorem guarantee a solution to $0 = x^2 + 6x + 10$ on the interval $[-1, 3]$?

6. Does the Intermediate Value Theorem guarantee a solution to $f(x) = 0$ for $f(x) = 2\sin(x) - 8\cos(x) - 3x^2$ on the interval $\left[0, \frac{\pi}{2}\right]$?

7. Verify that the IVT applies to this function on the indicated interval and find the value of c guaranteed by the theorem. $f(x) = x^2 - 3x + 1$ on the interval $[0, 6]$, $f(c) = 5$.

The Intermediate Value Theorem also helps us solve polynomial and rational inequalities.

Examples:

1. $(x+2)^2(3x-2)(x-1)^3 \leq 0$

2. $\dfrac{2x - 8x^2}{(x+1)^2} \geq 0$

3. $\dfrac{1}{x-1}+\dfrac{1}{x+2}<0$

4. $\dfrac{4}{x+1} - \dfrac{3}{x+2} \geq 1$

Why did we just work these problems?

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These inequalities are able to be solved because of the Intermediate Value Theorem (IVT). The IVT basically states that if $f(x)$ is continuous from $x = a$ to $x = b$, then you must pass through all points ($x =$ "c") plotted along the graph of $f(x)$.

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Note: Functions with complex roots do not meet the requirements of the IVT. Why??