Math 1431

Section 16679

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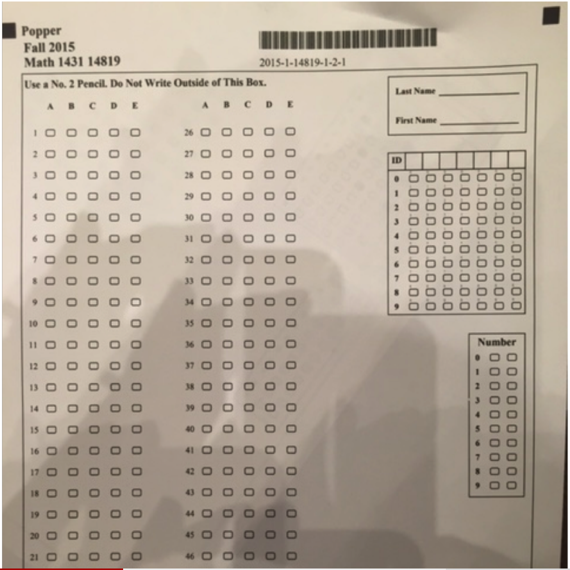
Questions

Popper 01

Poppers start today!



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1. Evaluate: $\lim_{x \to 9} \frac{x - 3}{\sqrt{x} - 3}$

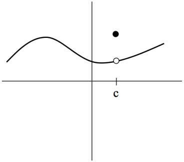
Continuity

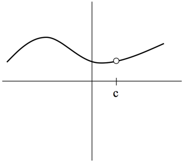
A function f is said to be continuous at a point c if

1. $f(c)$ is defined.
2. $\lim_{x \to c} f(x)$ exists.
3. $\lim_{x \to c} f(x) = f(c)$.

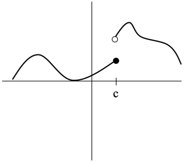
Types of discontinuity at a point

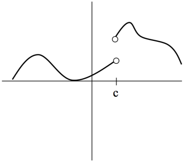
1. Removable:





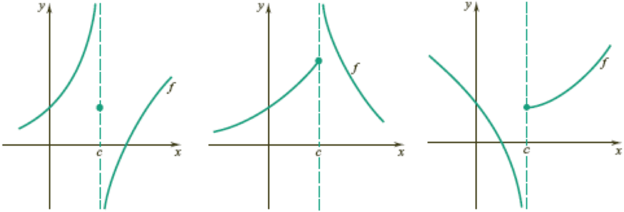
2. Non-Removable - Jump:





Types of discontinuity at a point

3. Non-Removable - Infinite:



5. $f(x) = \sqrt{x-3}$

6. $f(x) = \dfrac{\sqrt{x} - 1}{x^2 + 4x - 5}$

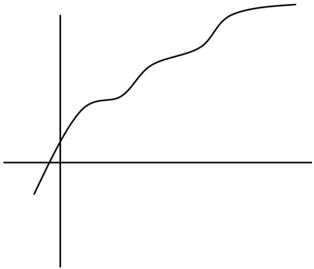
9. $g(x) = \begin{cases} x+2 & x < -2 \\ \sqrt{4-x^2} & -2 \le x < 2 \\ 1 & x = 2 \\ x-2 & x > 2 \end{cases}$

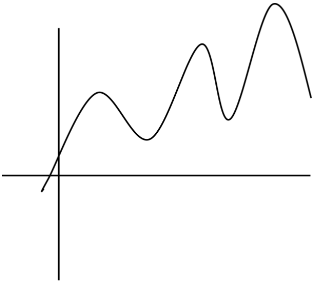
From "some more":

2. Discuss the continuity of $f(x) = \begin{cases} -x^2 & x < -1 \\ 3 & x = -1 \\ 2 - x & -1 < x \leq 1 \\ \frac{1}{x^2} & x > 1 \end{cases}$

A very important result of continuity is the **Intermediate Value Theorem**.

If $f(x)$ is continuous on the closed interval $[a, b]$ and K is a value between $f(a)$ and $f(b)$, then there is at least one value c in (a, b) such that $f(c) = K$.





Examples:
Use the intermediate value theorem to show that there is a solution to the given equation in the indicated interval.

1. $x^2 - 4x + 3 = 0$ on the interval $[2, 4]$

2. $x^3 - 6x^2 - x + 2 = 0$ on the interval $[0, 3]$

3. $2\tan(x) - x = 1$ on the interval $\left[0, \frac{\pi}{4}\right]$

4. Show there is a value of x between 1 and 3 so that $-3x^3 + 2x^4 = 7$

5. Does the Intermediate Value Theorem guarantee a solution to $0 = x^2 + 6x + 10$ on the interval $[-1, 3]$?

6. Does the Intermediate Value Theorem guarantee a solution to $f(x) = 0$ for $f(x) = 2\sin(x) - 8\cos(x) - 3x^2$ on the interval $\left[0, \frac{\pi}{2}\right]$?

7. Verify that the IVT applies to this function on the indicated interval and find the value of c guaranteed by the theorem. $f(x) = x^2 - 3x + 1$ on the interval $[0, 6]$, $f(c) = 5$.

Section 1.5 - The Intermediate Value Theorem

The Intermediate Value Theorem also helps us solve polynomial and rational inequalities.

Examples:

1. $(x+2)^2(3x-2)(x-1)^3 \leq 0$

Section 1.5 - The Intermediate Value Theorem

2. $\dfrac{2x - 8x^2}{(x+1)^2} \geq 0$

3. $\dfrac{1}{x-1} + \dfrac{1}{x+2} < 0$

4. $\dfrac{4}{x+1} - \dfrac{3}{x+2} \geq 1$

Why did we just work these problems?

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These inequalities are able to be solved because of the Intermediate Value Theorem (IVT). The IVT basically states that if $f(x)$ is continuous from $x = a$ to $x = b$, then you must pass through all points ($x =$ "c") plotted along the graph of $f(x)$.

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Note: Functions with complex roots do not meet the requirements of the IVT. Why??

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2. Does the IVT guarantee a solution to the equation $f(x) = \dfrac{x^2 - 2x + 1}{x - 1}$ on the interval $[0, 3]$.

Section 1.5 - The Extreme Value Theorem

The **Extreme Value Theorem** states:

If a function $f(x)$ is continuous on a closed interval $[a, b]$, then $f(x)$ has both a maximum and a minimum on $[a, b]$.

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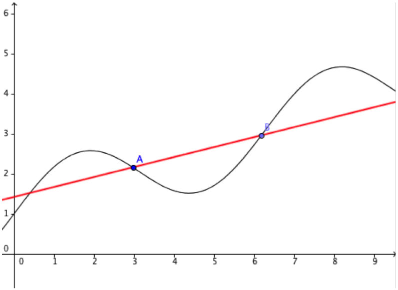
3. Evaluate the difference quotient $\left(\dfrac{f(x+h)-f(x)}{h}\right)$ for $f(x) = 2x + 3$ at $x = 1$.

We will be measuring how $f(x)$ changes when x changes but first, we need to understand slope a little more.

Slope of a secant line:

$$\frac{f(B) - f(A)}{B - A}$$



Also indicates the average rate of change over those values.

In calculus, we are more concerned with *instantaneous rate of change*, or rather the rate of change at a single point. To understand this, we will look at our secant line above and move A and B *very* close together - so close, the distance between A and B is near 0. Let's let the distance between A and B be h so we have $B = A + h$. Now our slope formula becomes:

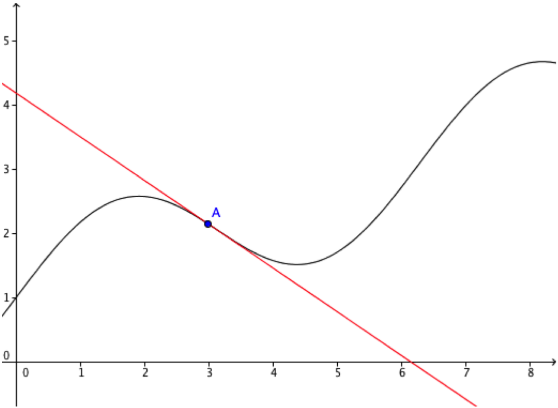
$$\frac{f(A+h) - f(A)}{(A+h) - A}$$

or rather,

$$\frac{f(A+h) - f(A)}{h}$$

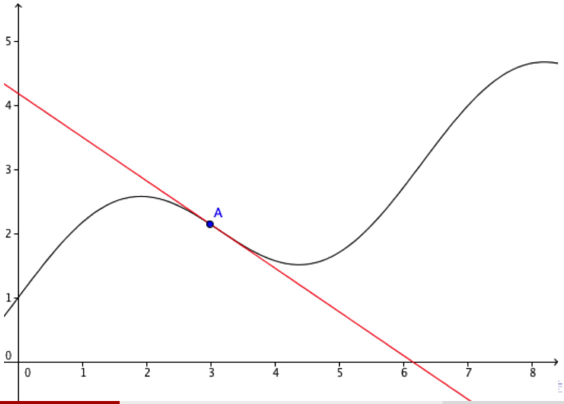
So, as this distance between A and B gets close to 0, we can say $h \to 0$.

So, as this distance between A and B gets close to 0, we can say $h \to 0$. Our graph will now look like this:



The slope of this *tangent line* at the point $x = A$ is the instantaneous rate of change at $x = A$. This is denoted with $f'(A)$ and is found by this formula:

$$f'(A) = \lim_{h \to 0} \frac{f(A+h) - f(A)}{h}$$



Examples:

1. Find the slope of the tangent line at $x = 3$ for $f(x) = x^2 + 1$

2. Find the slope of the tangent line at $x = 1$ for $f(x) = \sqrt{x}$

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4. The value of the derivative at $x = 2$ for $f(x) = \dfrac{1}{x+1}$ is the ____ of the tangent line at $x = 2$.

The Definition of Derivative

A function $f(x)$ is differentiable at x if and only if

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

exists. In this case, we denote

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

and we refer to $f'(x)$ as the derivative of f at x. $f'(x)$ can be thought of as the *slope function*. It gives the slope of the graph of $f(x)$ at any point x.

The derivative can also be denoted as:

$$f'(x), \;\; y', \;\; \frac{dy}{dx}, \;\; \frac{d}{dx}[f(x)]$$

3. Find the derivative of $f(x) = x^2 - 2x$ using the definition of the derivative.

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5. Find the slope of the tangent line to $f(x) = x^2 - 2x$ at $x = 3$.