Math 1431

Section 16679

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Questions

The Power Rule:

$$\frac{d}{dx}\left(x^n\right) = nx^{n-1},\ n \neq 0$$

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1. Find the derivative of $f(x) = \frac{1}{x} + \sqrt{x} + x$.

Higher Order Derivatives:

$$f'(x), \qquad f''(x), \qquad f'''(x), \qquad f^{(4)}(x)$$

$$\frac{d}{dx}f(x), \qquad \frac{d^2}{dx^2}f(x), \qquad \frac{d^3}{dx^3}f(x), \qquad \frac{d^4}{dx^4}f(x)$$

Examples:

1. $\dfrac{d^2}{dx^2}\left(3x^3 - 5x^2 + 2x - 1\right) =$

2. $\dfrac{d^3}{dx^3}\left(x^8 + 2x^5 - 2x + 5\right) =$

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2. Find $\dfrac{d^2}{dx^2}\left(\dfrac{2}{x} - x^5\right)$

Section 2.2 - Trig Derivatives

$$\frac{d}{dx}\sin(x) = \cos(x)$$
$$\frac{d}{dx}\cos(x) = -\sin(x)$$
$$\frac{d}{dx}\tan(x) = \sec^2(x)$$
$$\frac{d}{dx}\cot(x) = -\csc^2(x)$$
$$\frac{d}{dx}\sec(x) = \sec(x)\cdot\tan(x)$$
$$\frac{d}{dx}\csc(x) = -\csc(x)\cdot\cot(x)$$

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3. Given $f(x) = x^2 + \cos(x)$, find $f'(x)$.

The Product Rule:

If f and g are differentiable, then $f \cdot g$ is differentiable and

$$\frac{d}{dx}\left(f(x) \cdot g(x)\right) = f(x) \cdot g'(x) + f'(x) \cdot g(x)$$

Another way to write the Product Rule:

Suppose that $f(x) = u \cdot v$ where u and v are differentiable functions of x. Then,

$$f'(x) = u \cdot v' + u' \cdot v$$

or

$$f'(x) = u' \cdot v + u \cdot v'$$

Proof: Let $F(x) = f(x) \cdot g(x)$, then $F(x+h) = f(x+h) \cdot g(x+h)$ and

$$\begin{aligned}
F'(x) &= \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x) \cdot g(x)}{h} \\
&= \lim_{h \to 0} \frac{f(x+h) \cdot g(x+h) - f(x+h) \cdot g(x) + f(x+h) \cdot g(x) - f(x) \cdot g(x)}{h} \\
&= \lim_{h \to 0} \frac{f(x+h)[g(x+h) - g(x)] + [f(x+h) - f(x)]g(x)}{h} \\
&= \left[\lim_{h \to 0} f(x+h)\right] \cdot \left[\lim_{h \to 0} \frac{g(x+h) - g(x)}{h}\right] \\
&\qquad + \left[\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}\right] \cdot \left[\lim_{h \to 0} g(x)\right] \\
&= f(x) \cdot g'(x) + f'(x) \cdot g(x)
\end{aligned}$$

Examples: Find the derivative of each:

1. $y = (5x + 2)(x^2 + 1)$

2. $f(x) = (3x - 1)(2x^4 - x)$

3. $y = x^2 \cos(x)$

4. $f(x) = (x^2 - 2x + 1)\tan(x)$

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4. Find the derivative of $y = x^3 \cdot f(x)$.

The Quotient Rule:

If f and g are differentiable, then $\frac{f}{g}$ is differentiable (providing $g \neq 0$) and

$$\frac{d}{dx}\left(\frac{f(x)}{g(x)}\right) = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{\left(g(x)\right)^2}$$

Another way to write the QuotientRule:

Suppose that $f(x) = \frac{u}{v}$ where u and v are differentiable functions of x and $v \neq 0$. Then,

$$f'(x) = \frac{u' \cdot v - u \cdot v'}{v^2}$$

Examples: Find the derivative of each:

1. $y = \dfrac{x}{x^2 + 1}$

2. $f(x) = \dfrac{x^2 - 4}{x - 3}$

3. $y = \dfrac{1}{x+1}$

4. $\dfrac{d}{dx}\left(\dfrac{x}{\sin(x)}\right) =$

Quiz 7 Questions

5) Consider the function $f(x) = x^3 - 3x^2 + 3$. Find the points where the tangent line is horizontal.

Quiz 7 Questions

6) Given the function $f(x) = \frac{1}{3}x^3 - \frac{5}{2}x^2 + 4x + 2$, find the points where the tangent line has slope -2.

Quiz 7 Questions

9) Determine the number(s), x, between 0 and 2π where the line tangent to the function $f(x) = 4\sqrt{3}\sin(x) + 4\cos(x)$ is horizontal.

Motivation behind the chain rule:

Find the derivative of each:

1. $y = 5x^4$

2. $y = (2x + 1)^2$

3. $y = (2x + 1)^{14}$

Recall:

Composite functions are functions within functions.
They are written $f(g(x))$ or $(f \circ g)(x)$.

For example:
If $f(x) = 3x - 4$ and $g(x) = x^2$

,
then $f(g(x)) =$

and $g(f(x)) =$

To find the derivative of composite functions, we use the chain rule.

The Chain Rule:

Let $f(x)$ and $g(x)$ be separate functions of x and let $y = f(g(x))$, then

$$y' = f'(g(x)) \cdot g'(x)$$

or

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

Where in $\dfrac{dy}{du}$ we have substituted $y = f(x)$ and $u = g(x)$.

Examples: Find the derivative of each:

1. $y = (2x+1)^{14}$

2. $f(x) = (x-1)^5$

3. $y = (x^2 + 6x - 4)^3$

4. $g(x) = \sqrt{x^2 + 3}$

5. $y = 6(3x + 2)^4$

6. $g(x) = \sin^2(x)$

7. $f(x) = \left(x^2 + \frac{1}{x^2}\right)^3$

Section 2.3 - Differentiation Rules

8. $f(x) = \left(\dfrac{x}{2x^2+1}\right)^3$

9. $y = \cos\left(\sqrt{x}\right)$

10. Suppose $G(x) = f(h(x))$ with $h(1) = 2$, $f'(1) = 3$, $f'(2) = -6$, $h'(1) = 7$. Find $G'(1)$.

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5. Find $\frac{d}{dx}(f(g(x)))$.