Math 1431

Section 16679

Bekki George: rageorge@central.uh.edu

University of Houston

09/26/19

Office Hours: Tuesdays & Thursdays 11:45-12:45
(also available by appointment)
Office: 218C PGH

Course webpage: www.casa.uh.edu

Questions

Popper 08

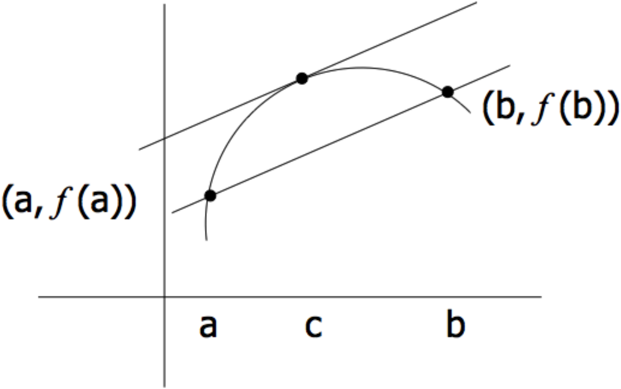
1. Find the value of c such that $f'(c) = 2$ when $f(x) = \sqrt{x} - x$.

If f is continuous on the closed interval $[a, b]$ and differentiable on (a, b), then there exists a number c in (a, b) such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

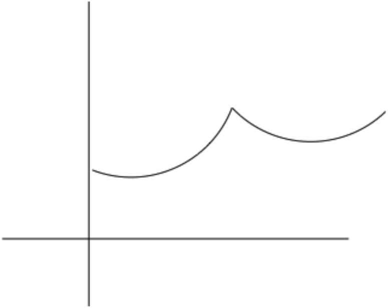
(slope of tangent line) = (slope of secant line)

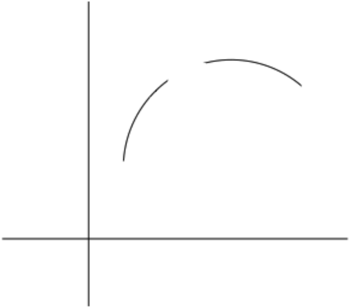
Geometrically, the theorem says that here is at least one point $(c, f(c))$ on f at which the tangent line is parallel to the secant line through the points $(a, f(a))$ and $(b, f(b))$.


(b, f (b))
(a, f (a))
a
c
b

Examples of when the MVT does not apply:

Examples of when the MVT does not apply:





Examples:

Determine if the MVT applies and if it does, find all values of c on the interval (a, b) such that $f'(c) = \frac{f(b)-f(a)}{b-a}$.

1. $f(x) = x^3 + x - 4$ on $[-1, 2]$

2. $f(x) = \sqrt{x}$ on $[0, 1]$

Section 3.2 - Mean Value Theorem

3. $f(x) = \dfrac{1}{x-1}$ on $[2, 5]$

Section 3.2 - Mean Value Theorem

4. $f(x) = x^{2/3}$ on $[-8, 8]$

4. $f(x) = x^{2/3}$ on $[1, 8]$

Popper 08

2. Find all value(s) of c (if any) that satisfy the conclusion of the Mean Value Theorem for the function $f(x) = \dfrac{1}{x+1}$ on the interval $[-2, 1]$.

5. $f(x) = \dfrac{1}{x+1}$ on $[0, 1]$

Section 3.2 - Rolle's Theorem

Rolle's Thm is a special case of the MVT. It states:

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f(a) = f(b)$ then there is at least one number c in (a, b) such that $f'(c) = 0$.

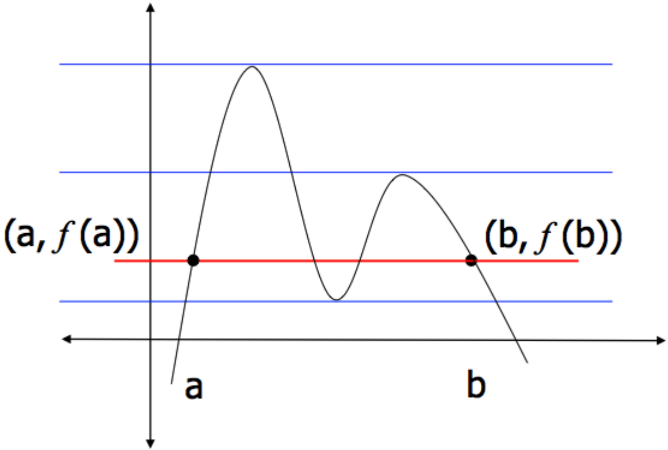
Section 3.2 - Rolle's Theorem

Rolle's Thm is a special case of the MVT. It states:

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f(a) = f(b)$ then there is at least one number c in (a, b) such that $f'(c) = 0$.

What does this mean?


(a, f (a))
(b, f (b))
a
b

Examples:

1. Find the two $x-$ intercepts of $f(x) = x^2 - x - 20$ and show using Rolle's theorem that $f'(x) = 0$ at some point between the two intercepts.

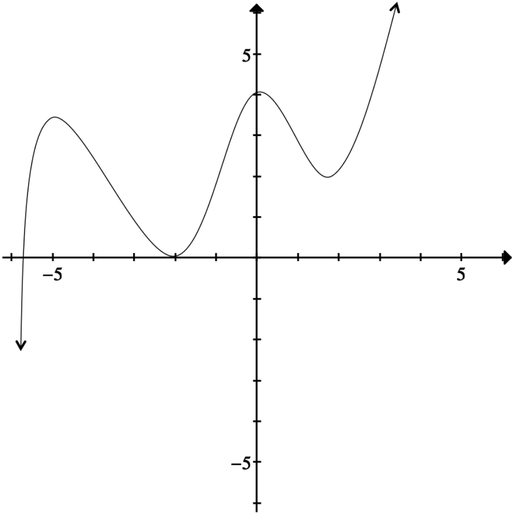
Popper 08

3. Decide whether Rolle's Theorem can be applied to $f(x) = x^2 + 3x$ on the interval [0,2]. If Rolle's Theorem can be applied, find all value(s) of c in the interval such that $f'(c) = 0$.

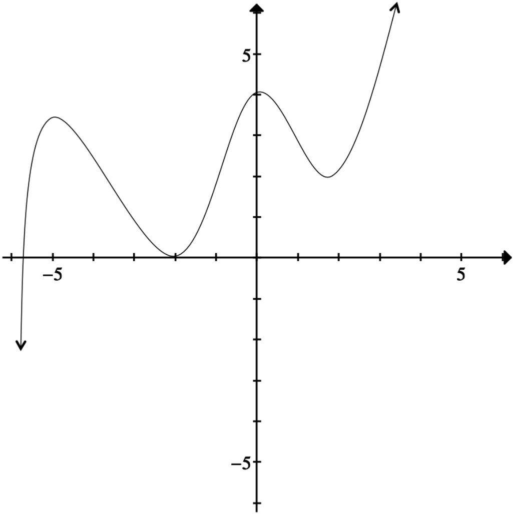
Popper 08

4. Which of the following functions fails to satisfy the conditions of The Mean Value Theorem on the given interval?

Intuitively, where is f increasing?



Intuitively, where is f decreasing?



Section 3.3 - Increasing and Decreasing Functions

In plain terms, a function is increasing if, as x moves to the right, its graph moves up, and is decreasing if its graph moves down.

A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

Since $f'(a)$ gives us the slope of the tangent line to $f(x)$ at $x = a$, it follows that:

where $f'(x)$ is positive, $f(x)$ is increasing

and

where $f'(x)$ is negative, $f(x)$ is decreasing.

In math terms

In math terms

f is increasing over an interval I
if and only if
$f(a) < f(b)$
for all $a, b \in I$ with $a < b$.

In math terms

$$f \text{ is increasing over an interval } I$$
$$\text{if and only if}$$
$$f(a) < f(b)$$
$$\text{for all } a, b \in I \text{ with } a < b.$$

Theorem: A function f is increasing on an interval I provided f is continuous and $f'(x) > 0$ at all but finitely many values in I.

And...

f is decreasing over an interval I
if and only if
$f(a) > f(b)$
for all $a, b \in I$ with $a < b$.

And...

f is decreasing over an interval I
if and only if
$f(a) > f(b)$
for all $a, b \in I$ with $a < b$.

Theorem: A function f is increasing on an interval I provided f is continuous and $f'(x) < 0$ at all but finitely many values in I.

Definition of Critical Number:

The numbers c in the domain of a function f for which either $f'(c) = 0$ or $f'(c)$ does not exist, are called the critical numbers of f.

The terms critical points and critical values are also used.

Examples:

1. Find the critical numbers of $f(x) = 3x^4 - 4x^3$.

2. Find the critical numbers of $f(x) = \dfrac{x-1}{x-3}$.

3. Find the critical numbers of $f(x) = (x^2 - 36)^{1/3}$.

Popper 08

5. Find all critical numbers: $f(x) = \dfrac{1}{x^2 - 4}$