Math 1431

Section 16679

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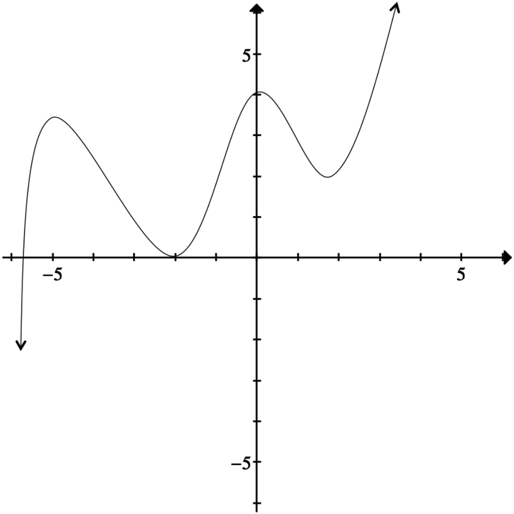
Questions

Intuitively, where is f decreasing?



In plain terms, a function is increasing if, as x moves to the right, its graph moves up, and is decreasing if its graph moves down.

A function is strictly monotonic on an interval if it is either increasing on the entire interval or decreasing on the entire interval.

Since $f'(a)$ gives us the slope of the tangent line to $f(x)$ at $x = a$, it follows that:

where $f'(x)$ is positive, $f(x)$ is increasing

and

where $f'(x)$ is negative, $f(x)$ is decreasing.

In math terms

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$$\text{if and only if}$$
$$f(a) < f(b)$$
$$\text{for all } a, b \in I \text{ with } a < b.$$

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Theorem: A function f is increasing on an interval I provided f is continuous and $f'(x) > 0$ at all but finitely many values in I.

And...

f is decreasing over an interval I
if and only if
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for all $a, b \in I$ with $a < b$.

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Theorem: A function f is increasing on an interval I provided f is continuous and $f'(x) < 0$ at all but finitely many values in I.

Definition of Critical Number:

The numbers c in the domain of a function f for which either $f'(c) = 0$ or $f'(c)$ does not exist, are called the critical numbers of f.

The terms critical points and critical values are also used.

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1. Find all critical numbers for $f(x) = \left(9 - x^2\right)^{3/5}$

Theorem: Test for Increasing or Decreasing Functions

Let f be a function that is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

1. If $f'(x) > 0$ for all x in (a, b), then f is increasing on $[a, b]$.
2. If $f'(x) < 0$ for all x in (a, b), then f is decreasing on $[a, b]$.
3. If $f'(x) = 0$ for all x in (a, b), then f is constant on $[a, b]$.

To find intervals on which a continuous function is increasing or decreasing:

1. Locate the critical numbers to determine test intervals.
2. Determine the sign of $f'(x)$ at one value in each interval.
3. Using the previous theorem, determine if the function is increasing or decreasing on the interval.

Examples: Determine the intervals of increase and/or decrease for each of the following.

1. $f(x) = \frac{1}{3}x^3 + \frac{5}{2}x^2 + 6x - 3.$

2. $f(x) = \dfrac{x^2 + 1}{x^2 - 1}$

3. $f(x) = \dfrac{2x}{x^2 - 4}$

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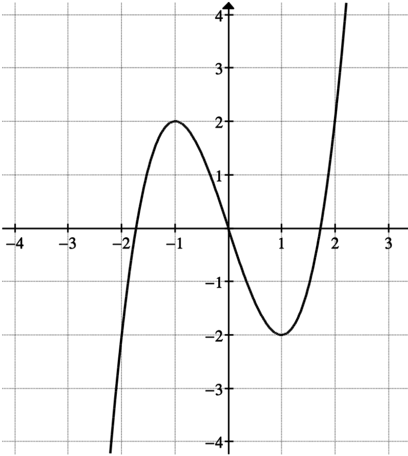
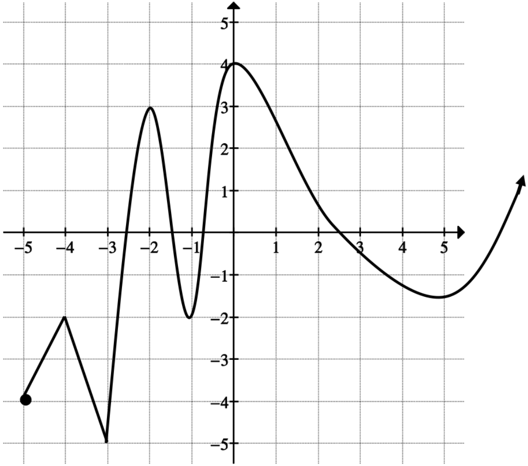
2. If this is the graph of $f'(x)$, how many critical numbers are there?



Where are the local "extremes" of the function shown? What can be said about the values of the derivatives at those points?



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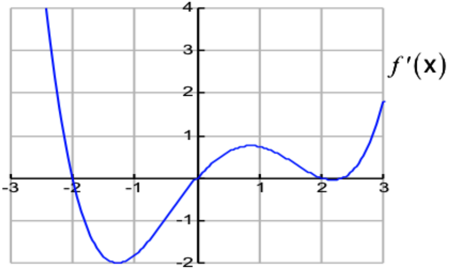
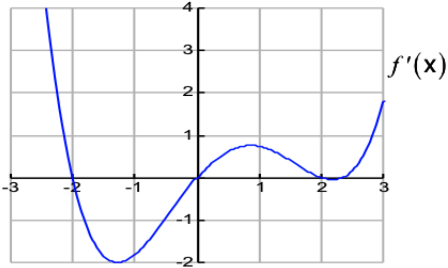
3. The graph of $f'(x)$ is given below. Where is $f(x)$ increasing?



The graph of $f'(x)$ is shown. Give a possible sketch for $f(x)$.



How can we classify critical points as either local (or relative) maximums or local minimums?

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First Derivative Test:

Let c be a critical number of a function f that is continuous on an open interval I containing c.
If f is differentiable on the interval, except possibly at c, then $f(c)$ can be classified as follows:

1. If $f'(x)$ changes from negative to positive at c, then $f(c)$ is a local minimum of f.
2. If $f'(x)$ changes from positive to negative at c, then $f(c)$ is a local maximum of f.

When using the First Derivative Test, be sure to consider the domain of the function. The $x-$value where the function is undefined must be used with the critical numbers to determine the test intervals.

If $f' > 0$, what conclusion can be made about f?

If $f' < 0$, what conclusion can be made about f?

If $f'' > 0$, what conclusion can be made about f'?

If $f'' < 0$, what conclusion can be made about f'?

So, if $f'(c) = 0$ and $f''(c) > 0$, what conclusion can be made about $f(c)$?

And, if $f'(c) = 0$ and $f''(c) < 0$, what conclusion can be made about $f(c)$?

Thm: **The Second Derivative Test**

Let f be a function such that $f'(c) = 0$ and the second derivative of f exists on an open interval containing c.

1. If $f''(c) > 0$, then $f(c)$ is a local minimum.
2. If $f''(c) < 0$, then $f(c)$ is a local maximum.
3. If $f''(c) = 0$, then the test fails. In such cases, you can use the First Derivative Test.

How to use the Second Derivative Test:

- Determine the critical numbers using the first derivative.
- Plug these numbers into the second derivative and get the value.
- If the value is positive, you have a relative minimum.
- If the value is negative, you have a relative maximum.
- If the value is zero, use the First Derivative Test to determine if there is a local max or min.

Examples:

1. Locate the local extrema for $f(x) = 2x^3 + 3x^2 - 12x$

2. Locate the local extrema for $f(x) = 2\sin(x) + \cos(2x)$ on $(0, 2\pi)$

3. Find the critical numbers, the intervals on which the function is increasing or decreasing, and all local extrema.
   $f(x) = -(x-1)^2(x+2)$

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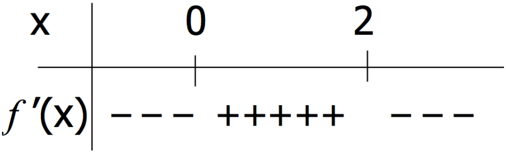
4. A slope chart is shown below for the function f. Classify the critical point at $x = 0$.



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5. A slope chart is shown below for the function f. Classify the critical point at $x = 2$.