Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

7. Determine if events A and B are independent. $P(A) \cdot P(B) = P(A \cap B)$

a. $P(A) = 0.9,\ P(B) = 0.3,\ P(A \cap B) = 0.27$

$(.9)(.3) = .27$ yes

b. $P(A) = 0.4,\ P(B) = 0.6,\ P(A \cap B) = 0.20$

$(.4)(.6) = .24 \neq .20$ not ind.

independent $\neq$ mutually excl.
disjoint
$P(A \cap B) = 0$

$AND = \cap$

$OR = \cup$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Math 2311

Class Notes for More Probability Review and Section 3.1

#21 from text: F = felony C = college $P(F) = .3$

Thirty percent of the students at a local high school face a disciplinary action of some kind before they graduate. Of those "felony" students, 40% go on to college. Of the ones who do not face a disciplinary action, 60% go on to college.

$P(C|F^c) = .6$ $P(C|F) = .4$

a. What is the probability that a randomly selected student both faced a disciplinary action and went on to college? $P(F \cap C)$

$$P(C|F) = \frac{P(C \cap F)}{P(F)} = \frac{P(C \cap F)}{.3}$$

$$.4 = \frac{P(C \cap F)}{.3}$$

$$.12 = P(C \cap F)$$

b. What percent of the students from the high school go on to college?

$$P(C) = P(C \cap F) + P(C \cap F^c)$$

$$= .12 + .42 = \boxed{.54}$$

c. Show if events {faced disciplinary action} and {went to college} are independent or not.

$$P(C) \cdot P(F) = (.54)(.3) = .162 \neq .12$$

not independent.

$P(C \cap F^c)$

$$P(C|F^c) = \frac{\boxed{P(C \cap F^c)}}{P(F^c)}$$

$P(F) = .3$

$P(F^c) = .7$

$$.6 = \frac{x}{.7}$$

$$\boxed{.42 = x}$$

Popper 02

$P(A) = 0.73$, $P(B) = 0.44$, $P(A \cup B) = 0.89$

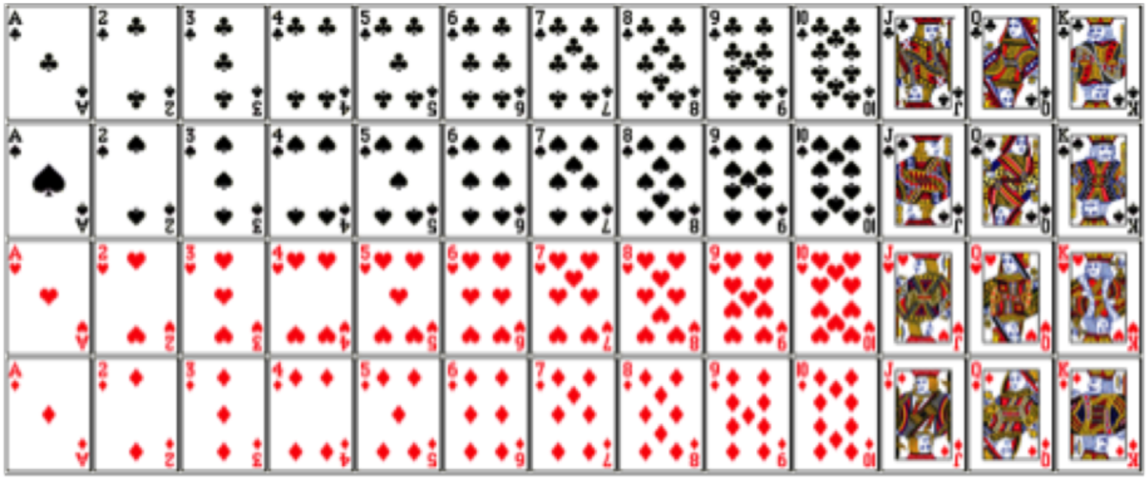
1. $P(A \cap B) =$
    a. 0.3212
    b. 0.2800
    c. 0.3836
    d. 0.6364

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

2. $P(A \mid B) =$
    a. 0.3212
    b. 0.2800
    c. 0.3836
    d. 0.6364

$\frac{P(A \cap B)}{P(B)}$

Suppose you are playing poker with a standard deck of 52 cards:



How many 5 card hands are possible?

$${}_{52}C_5 = 2{,}598{,}960$$

How many ways can you get 4 kings in a hand?

$${}_4C_4 \quad \underbrace{K\,K\,K\,K}_{1}\ \underset{48}{\boxed{}} = 48$$

How many ways can you have any 4 of a kind hand?

$$13\,({}_4C_4)\cdot 48 = 624$$

What is the probability of getting 4 of a kind? $P(4 \text{ of kind}) = \dfrac{624}{2598960} \approx .0002$

How many ways can you have 3 kings and 2 fives?

$$\underbrace{K\,K\,K}_{{}_4C_3} \cdot \underbrace{5\,5}_{{}_4C_2} = 4 \cdot 6 = 24$$

How many ways can you get a full house?

3 of a kind + 2 of a kind

$$13({}_4C_3) \cdot 12({}_4C_2) = 3744$$

What is the probability of getting a full house?

$$P(\text{full house}) = \frac{3744}{2598960} = .0014$$

Problems from Quiz 2:

A researcher randomly selects 2 fish from among 10 fish in a tank and puts each of the 2 selected fish into different containers. How many ways can this be done?

ORDER MATTERS! nP_r

10 9

$$_{10}P_2 = \frac{10!}{(10-2)!} = 90$$

An experimenter is randomly sampling 4 objects in order from among 61 objects. What is the total number of samples in the sample space?

Since this says "in order", this is a Permutation. the answer is 61P4 = 12524520

$_{61}C_4$

This is if it didn't say "in order"

Choose (61,4) = 521855

How many license plates can be made using 3 digits and 4 letters if repeated digits and letters are not allowed?

10 26

$$_{10}P_3 \cdot {}_{26}P_4 = 258,336,000$$

123 ABCD

132 ACDB

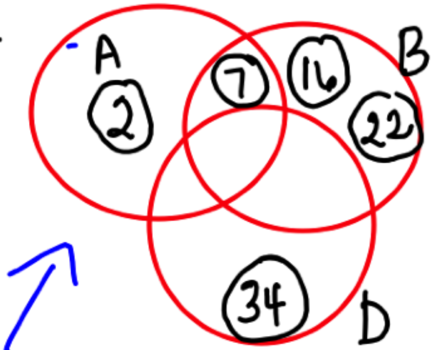
Perm.

$\underline{10}\ \underline{9}\ \underline{8}\ \ \underline{26}\ \underline{25}\ \underline{24}\ \underline{23}$

$$\underbrace{10 \cdot 10 \cdot 10 \cdot 26 \cdot 26 \cdot 26 \cdot 26}_{\text{repeats are allowed}}$$

DeMorgan's Law $(E\cap F)^c = E^c \cup F^c$ $(E \cup F)^c = E^c \cap F^c$

Let A = {2, 7}, B = {7, 16, 22}, D = {34} and S = sample space = A ∪ B ∪ D. Find $\boxed{(A^c \cap B^c)^c}$.



$A^c = \{16, 22, 34\}$

$\boxed{B^c = \{2, 34\}}$

$A^c \cap B^c = 34$

$(A^c \cap B^c)^c = \{2, 7, 16, 22\}$

$(A^c)^c \cup (B^c)^c$

$\downarrow$

$A \cup B$

$\downarrow$

Popper 02

3. Let A = {2, 7}, B = {7, 16, 22}, D = {34} and S = sample space = A ∪ B ∪ D. Identify $B^c \cup A$.

a) {2, 7, 16, 22}

b) {2, 16, 22, 34}

c) {2, 7, 34}

d) {2, 34}

e) {2, 7}

$B^c \cap A = \{2\}$

58 do have cracks

In a shipment of 71 vials, only 13 do not have hairline cracks. If you randomly select one vial from the shipment, what is the probability that it has a hairline crack?

$$P(\text{crack}) = \frac{58}{71}$$

Popper 02

62-14 = 48 have no crack

4. In a shipment of 62 vials, only 14 do not have hairline cracks. If you randomly select one vial from the shipment, what is the probability that it has a hairline crack?

a) $\frac{1}{14}$
b) $\frac{24}{31}$
c) $\frac{7}{24}$
d) $\frac{7}{31}$
e) $\frac{1}{62}$

(48C1)/(62C1)

38 do have cracks

In a shipment of 54 vials, only 16 do not have hairline cracks. If you randomly select 3 vials from the shipment, what is the probability that none of the 3 vials have hairline cracks?

$$\frac{{}_{16}C_3}{{}_{54}C_3} = \frac{560}{24804}$$

exactly 2 have cracks:

$$\frac{{}_{38}C_2 \cdot {}_{16}C_1}{{}_{54}C_3} = \frac{703 \cdot 16}{24804} \approx .453$$

The probability that a randomly selected person has high blood pressure (the event H) is P(H) = 0.4 and the probability that a randomly selected person is a runner (the event R) is P(R) = 0.3. The probability that a randomly selected person has high blood pressure and is a runner is 0.2. Find the probability that a randomly selected person either has high blood pressure or is a runner or both.

$P(H \cap R) = .2$

$$P(H \cup R) = P(H) + P(R) - P(H \cap R)$$
$$= .4 + .3 - .2$$
$$= .5$$

Popper 02

$P(H \cap R) = .2 \qquad P(H \cup R) = \underline{\underline{.5}}$

5. The probability that a randomly selected person has high blood pressure (the event H) is $\boxed{P(H) = 0.4}$ and the probability that a randomly selected person is a runner (the event R) is $\boxed{P(R) = 0.3.}$ The probability that a randomly selected person has high blood pressure and is a runner is 0.2. Find the probability that a randomly selected person has high blood pressure and is not a runner.

$P(H \cap R^c)$

a) 0.5
b) 0.2
c) 0.7
d) 0.6
e) 0.4

Are events H and R independent? NO *Mutually exclusive?* NO

$P(H \cap R) \neq 0$

$P(H)\,P(R) = (.4)(.3) = .12 \neq .2$

$P(H \cap O) = .08$ $P(H) = .16$ $P(O) = .26$

Hospital records show that 16% of all patients are admitted for heart disease, 26% are admitted for cancer (oncology) treatment, and 8% receive both coronary and oncology care. What is the probability that a randomly selected patient is admitted for coronary care, oncology or both? (Note that heart disease is a coronary care issue.)

$$P(H \cup O) = .16 + .26 - .08$$

$$= .34$$

What is the probability that a randomly selected patient is admitted for something other than coronary care?

$$P(H^c) = .84$$

$$1 - .16$$

Popper 02

8 function

6. Among 9 electrical components exactly one is known not to function properly. If 3 components are randomly selected, find the probability that all selected components function properly.

a) 2/3
b) 1/3
c) 8/9
d) 5/9
e) 1

$$\frac{{}_8C_3}{{}_9C_3}$$

What is the probability that exactly one does not function properly?

$$\frac{{}_1C_1 \; {}_8C_2}{{}_9C_3} = \frac{1}{3}$$

What is the probability that at least one does not function properly?

1 or more

1 or 2 or 3

$1/3 + 0 + 0 = 1/3$

Section 3.1

R.V. .

A **random variable** is a variable whose value is a numerical outcome of a random phenomenon. It assigns one and only one numerical value to each point in the sample space for a random experiment.

A $\boxed{\textbf{discrete random variable}}$ is one that can assume a countable number of possible values
A $\boxed{\textbf{continuous random variable}}$ can assume any value in an interval on the number line.

A **probability distribution table of *X*** consists of all possible values of a discrete random variable with their corresponding probabilities.

Example: Suppose a family has 3 children. Show all possible gender combinations:

set $\left\{ \begin{matrix} GBG, & GGG, & BBB, & BBG \\ BGG, & GGB, & GBB, & BGB \end{matrix} \right\} 8$ $\quad \underline{2} \cdot \underline{2} \cdot \underline{2} = 8$

Now suppose we want the probability distribution for the number of girls in the family.

discrete RV $\Rightarrow X = \#$ of girls

3 or 2 or 1 or 0

Draw a probability distribution table for this example.

X	0	1	2	3
$P(X)$	$1/8$	$3/8$	$3/8$	$1/8$

$= 1 \quad (8/8)$

Find $P(X > 2)$

$= P(X = 3) = 1/8$

$P(X < 1)$

$= 1/8$

$P(1 < X \leq 3) = 3/8 + 1/8 = 1/2$

mean

The mean, or $\boxed{\textbf{expected value}}$, of a random variable X is found with the following formula

$$\mu_X = E[X] = x_1 p_1 + x_2 p_2 + \cdots + x_n p_n$$

What is the expected number of girls in the family above?

$$E[X] = 0\left(1/8\right) + 1\left(3/8\right) + 2\left(3/8\right) + 3\left(1/8\right) = 3/2$$

The variance of a random variable X can be found using the following:

$$\Rightarrow \sigma_X^2 = Var[X] = (x_1 - \mu_X)^2 p_1 + (x_2 - \mu_X)^2 p_2 + \cdots + (x_n - \mu_X)^2 p_n$$

$$= \sum (x_i - \mu_X)^2 p_i$$

An alternate formula is:

$$\boxed{\sigma_X^2 = Var[X] = E[X^2] - (E[X])^2}$$

Find the **standard deviation** for the number of girls in the example above.

x	0	1	2	3
$\Rightarrow x^2$	0	1	4	9
$P(x)$	$1/8$	$3/8$	$3/8$	$1/8$

$$E[X^2] = 0(1/8) + 1(3/8) + 4(3/8) + 9(1/8) = 3$$

$$VAR = 3 - (3/2)^2 = 3/4$$

Popper 02

$$1 = \frac{1}{25} + \frac{3}{50} + \frac{1}{20} + \frac{1}{100} + X$$

Given the following sampling distribution:

X	-18	-14	2	11	20
P(X)	1/25	3/50	1/20	1/100	___

7. What is P(X=20)?
   a. 16/100
   b. 84/100
   c. 53/100
   d. none of these

8. What is P(X>2)? $P(X=11) + P(X=20)$
   a. 24/25
   b. 9/10
   c. 85/100
   d. none of these

9 & 10 are A.