Math 2311

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Office Hours: MW 11am to 12:45pm in 639 PGH

Online Thursdays 4-5:30pm

And by appointment

Class webpage: http://www.math.uh.edu/~bekki/Math2311.html

So far ….

Hypothesis Testing:

- One sample:
  - T-test – testing the mean and population standard deviation is unknown.
  - Matched Pairs t-test – dependent data (before and after or pre, post information). We are testing the mean difference (subtract first). Usually the null hypothesis is $\mu_D = 0$, meaning no change.
  - Z-test – testing the mean and the population standard deviation is known
    – testing proportions
- Two (or more) samples: this week

Steps:

- Check conditions
- State the null and alternate hypothesis
- Sketch rejection region
- Find test statistic
- Get the p-value
- State your conclusion

Another example on Errors:

Suppose your doctor has just informed you that a lump has been discovered in your left foot which may or may not be a benign growth. You have only two choices: to wait and see if the lump spreads or remove your entire left foot. What would you do?

	H_0 is true (benign)	H_0 is false (the lump spreads)
Fail to reject H_0 (wait and see what happens)	OK	lumpy Type II
Reject H_0 (remove your left foot)	hop (no foot) Type I	OK made right choices

Type I error – reject H_0 when H_0 is true
Type II error – fail to reject H_0 when H_0 is false

Q12

Question 11

Matched pairs

You did not answer the question.

An auditor for a hardware store chain wished to compare the efficiency of two different auditing techniques. To do this he selected a sample of nine store accounts and applied auditing techniques A and B to each of the nine accounts selected. The number of errors found in each of techniques A and B is listed in the table below:

accts	Errors in A	Errors in B	Diff A-B
1	27	13	14
2	30	19	11
3	28	21	
	30	19	
	34	36	
	32	27	
	31	31	
	22	23	
	27	32	

← find $\bar{x}$ for this

Does the data provide sufficient evidence to conclude that the number of errors in auditing technique A is greater than the number of errors in auditing technique B at the 0.1 level of significance? Select the [Alternative Hypothesis, Value of the Test Statistic]. (Hint: the samples are dependent)

$H_0: \mu_D = 0$

$H_a: \mu_D > 0$

8.3 – Comparing Two Means

Two – sample *t* – tests compare the responses to two treatments or characteristics of two populations. There is a separate sample from each treatment or population. These tests are quite different than the matched pairs *t* – test discussed in section 8.1.

How can we tell the difference between dependent and independent populations/samples?

If you rearrange one list then it doesn't match up (doesn't make sense) ⇒ dependent (matched pairs)

The null and alternate hypotheses would be:

$H_0: \mu_1 = \mu_2$ $\quad$ $H_0: \mu_1 = \mu_2$ $\quad$ $H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$ or $H_a: \mu_1 < \mu_2$ or $H_a: \mu_1 \neq \mu_2$

df = smaller n − 1

And the assumptions for a two-sample *t* – test are:

1. We have two independent SRSs, from two distinct populations and we measure the same variable for both samples.
2. Both populations are normally distributed with unknown means and standard deviations. (Or if each given sample size is greater than or equal to 30.)

Two-sample t – test statistic:

$$t = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$

← if $H_0: \mu_1 = \mu_2$
then this is 0

The degrees of freedom is equal to the smaller of $n_1 - 1$ and $n_2 - 1$.

Example:

1. The president of an all-female school stated in an interview that she was sure that the students at her school studied more, on average, than the students at a neighboring all-male school. The president of the all-male school responded that he thought the mean study time for each student body was undoubtedly about the same and suggested that a study be undertaken to clear up the controversy. Accordingly, independent samples were taken at the two schools with the following results:

School	Sample Size	Mean Study Time (hrs)	Standard deviation (hrs)
All Female	65 n_F	18.56	4.35
All Male	75 n_m	17.95 $\bar{X}_m$	4.87 S_m

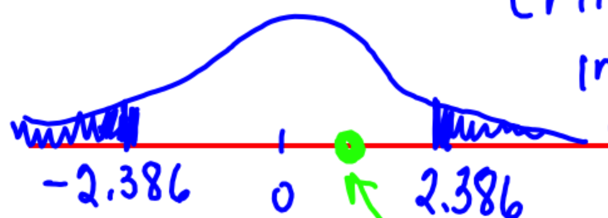
$\bar{X}_F = 18.56 \quad S_F = 4.35$

Determine, at the 2% level of significance, if there is a significant difference between the mean studying times of the students in the two schools based on these samples.

$\alpha = .02$

critical value

invT(.01, 64)



$H_0: \mu_F = \mu_m$

$H_a: \mu_F \neq \mu_m$

test statistic: $t = \dfrac{(18.56 - 17.95)}{\sqrt{\dfrac{4.35^2}{65} + \dfrac{4.87^2}{75}}} = \boxed{.7827}$

pvalue: $2\,p(t > .7827)$

$= 2 \cdot \text{tcdf}(.7827, 999999, 64)$

$2(1 - \text{pt}(.7827, 64))$ ← RStudio

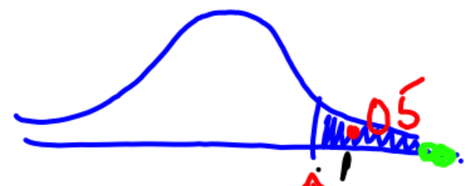
α

$= .437 > .02$

Based on 2% sig. level we will fail to reject the null hypothesis which states there is no difference in the mean study times for the two schools.

3. A study was conducted to determine whether remediation in basic mathematics enabled students to be more successful in an elementary statistics course. Samples of final exam scores were taken from students who had remediation and from students who did not. Here are the results of the study:

	Remedial ①	Non-remedial ②
Sample size	100 n_1	40 n_2
Mean Exam Grade	83.0 $\bar{x}_1$	76.5 $\bar{x}_2$
Std Dev for Exam	2.76 s_1	4.11 s_2



invT(.95, 39)

1.685

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 > \mu_2$

$\alpha = .05$

Test, at the 5% level, whether the remediation helped the students to be more successful.

$$t = \frac{(83-76.5)}{\sqrt{\frac{2.76^2}{100} + \frac{4.11^2}{40}}} = 9.206$$

pvalue

$p(t > 9.206)$ = tcdf(9.206, big, 39)

1 − pt(9.206, 39)

$\approx 0 \quad (1.26 \times 10^{-11})$

$0 < .05$

Reject H_0

Popper 21

1. A *t* test is used instead of a *z* test because
   a. the population mean is unknown.
   b. the population size is unknown.
   c. the population standard deviation is unknown.
   d. the population standard deviation is known.
   e. the population mean is known.

$\alpha = .05$

2. Suppose a test of the hypotheses produces a P-value of 0.15. The correct action would be to

   $.15 > .05$

   a. Reject H_o
   b. Accept H_o
   c. Fail to reject H_o
   d. Accept H_a

pvalue $< \alpha$
Reject H_0

$H_0: \mu = \#$

$H_a: \mu \,\square\, \#$

$> \; < \; \neq$

$H_0: \mu_1 = \mu_2$

$H_a: \mu_1 \,\square\, \mu_2$

8.4 – Comparing Two Proportions

When comparing two population proportions in an inference test, we use a **two-sample z test** for the proportions.

The null and alternate hypotheses would be:

$H_0 : p_1 = p_2$ $\quad$ $H_0 : p_1 = p_2$ $\quad$ $H_0 : p_1 = p_2$

$H_a : p_1 > p_2$ or $H_a : p_1 < p_2$ or $H_a : p_1 \neq p_2$

The assumptions are the same as for a confidence interval for the difference of two proportions:

1. Both samples must be independent SRSs from the populations of interest.
2. The population sizes are both at least ten times the sizes of the samples.
3. The number of successes and failures in both samples must all be ≥ 10.

$H_0 : p_1 = p_2$ then this is 0

And the test statistic is:

$$z = \frac{\hat{p}_1 - \hat{p}_2 - \underbrace{(p_1 - p_2)}}{\sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}}$$

If p_1 and p_2 are unknown, we will use $\hat{p}_1$ and $\hat{p}_2$ to approximate standard deviation. When we substitute $\hat{p}_1$ and $\hat{p}_2$ into standard deviation "formula," this gives us the standard error of

$$\sqrt{\frac{\hat{p}_1(1-\hat{p}_1)}{n_1}+\frac{\hat{p}_2(1-\hat{p}_2)}{n_2}}.$$

Example:

1. Is the proportion of left-handed students higher in honors classes than in academic classes? Two hundred academic and one hundred honors students from grades 6-12 were selected throughout a school district and their left or right handedness was recorded. The sample information is:

	Honors (1)	Academic (2)
Sample size n_1, n_2	100	200
Number of left-handed students	18	32

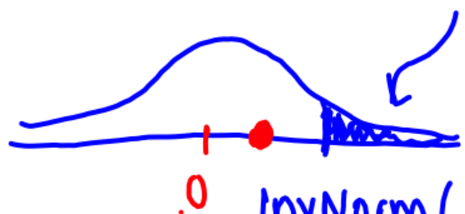
$\hat{p}_1 = 18/100 = .18$

$\hat{p}_2 = 32/200 = .16$

Is there sufficient evidence at the 5% significance level to conclude that the proportion of left-handed students is greater in honors classes? $\alpha = .05$

$H_0: p_1 = p_2$

$H_a: p_1 > p_2$



InvNorm(.95)
1.645

$$z = \frac{.18 - .16}{\sqrt{\frac{.18(1-.18)}{100} + \frac{.16(1-.16)}{200}}} = 0.432$$

pvalue

$p(z > .432) = .333$

$.333 > \underset{}{\overset{\alpha}{.05}}$

Fail to reject H_0

Another example:

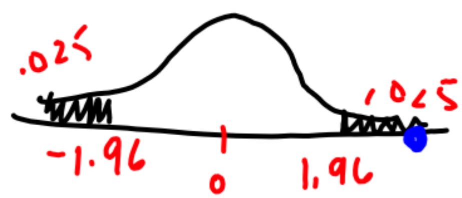
North Carolina State University looked at the factors that affect the success of students in a required chemical engineering course. Students must get a C or better in the course in order to continue as chemical engineering majors. There were 65 students from urban or suburban backgrounds, and 52 of these students succeeded. Another 55 students were from rural or small-town backgrounds; 30 of these students succeeded in the course. Test the claim to see if there is a difference between the urban and suburban success rates at the 5% level. $\alpha = .05$

1: urban/suburban $\quad \hat{p}_1 = 52/65 \quad n_1 = 65$

2: rural/smalltown $\quad \hat{p}_2 = 30/55 \quad n_2 = 55$

$H_0: p_1 = p_2$

$H_a: p_1 \neq p_2$



$$z = \frac{52/65 - 30/55}{\sqrt{\frac{52/65(1-52/65)}{65} + \frac{30/55(1-30/55)}{55}}} = 3.049$$

pvalue

$2P(z > 3.049)$

2normalcdf(3.049, big)

2(1-pnorm(3.049))

$.0023 < \alpha = .05$

Reject H_0

Popper 21

3. If we have a two-tailed test (not equal Ha), we should do what to find the p-value?
    a. Say its 0
    b. If the test statistic is negative, use $2 \cdot p(z \text{ or } t < test\ statistic)$ and if the test statistic is positive, use $2 \cdot p(z \text{ or } t > test\ statistic)$
    c. If the test statistic is negative, use $2 \cdot p(z \text{ or } t > test\ statistic)$ and if the test statistic is positive, use $2 \cdot p(z \text{ or } t < test\ statistic)$

4. Suppose a test of the hypotheses produces a P-value of 0.001. The correct action would be to
    a. Reject H_o
    b. Accept H_o
    c. Fail to reject H_0
    d. Accept H_a

5. Choose A