Density Curves

Section 4.1

Cathy Poliak, Ph.D.
cathy@math.uh.edu
Office hours: T Th 2:30 pm - 5:15 pm 620 PGH

Department of Mathematics
University of Houston

Februrary 16, 2016

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Outline

Popper Set Up

- Fill in all of the proper bubbles.
- Use a #2 pencil.
- This is popper number 05.

Popper Questions

1. A sample is the set of all possible data values for a given subject under consideration.

   a. True   b. False

2. X is the number of days it rained last month where you lived. X is an example of a discrete random variable.

   a. True   b. False

3. X is the amount of rainfall in your state last month. X is an example of a continuous random variable.

   a. True   b. False

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Probability Distribution

The **probability distribution** of a random variable X tells us what values X can take and how to assign probabilities to those values. Requirements for a probability distribution:

1. The sum of all the probabilities equal 1.

2. The probabilities are between 0 and 1, including 0 and 1.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Continuous Probability Distribution

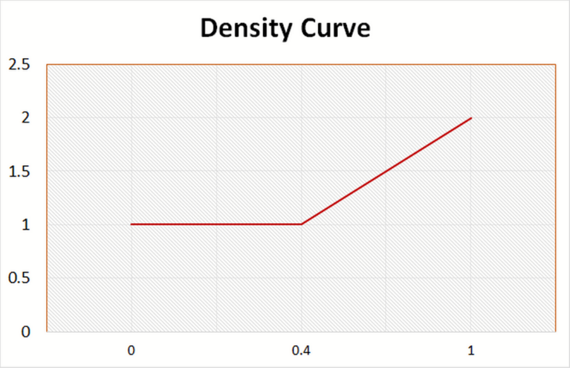
- The probability distribution of a continuous random variable X is described by a density curve.
- The probability of any event is the area under the density curve and above the values of X that makes up the event.
- The mean is the center or expected value of that distribution.
- The standard deviation is the spread of that distribution.

Density Curves

- A **mathematical model** for a probability distribution of a continuous random variable.
- This curve is always on or above the horizontal axis.
- The area under a density curve is exactly 1.
- The **area** under the curve and between any range of values is the probability that an observation falls in that range.

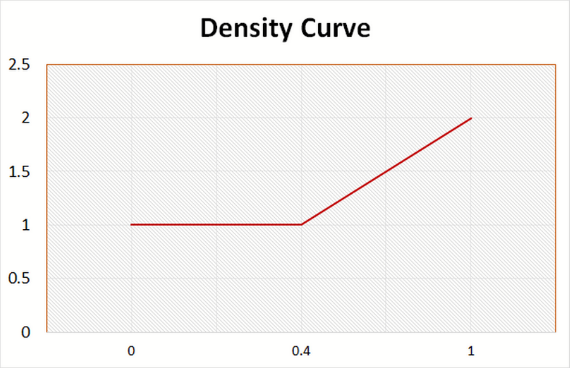
Example of Density Curve

The following graph is an example of a density curve that consists of two line segments. The first goes from the point (0,1) to the point (0.4,1). The second goes from (0.4,1) to (0.8,2) in the xy-plane. Does this meet the requirements of a probability distribution?

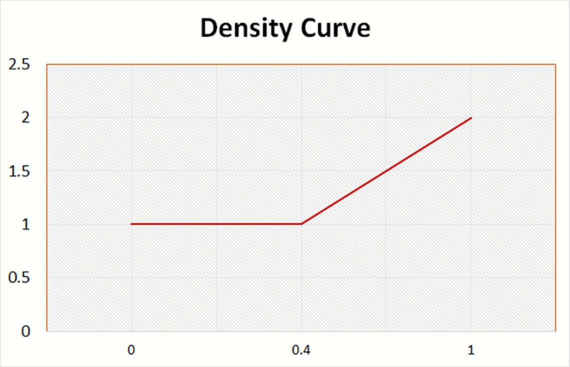


UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

What percent of the observations fall below 0.4?

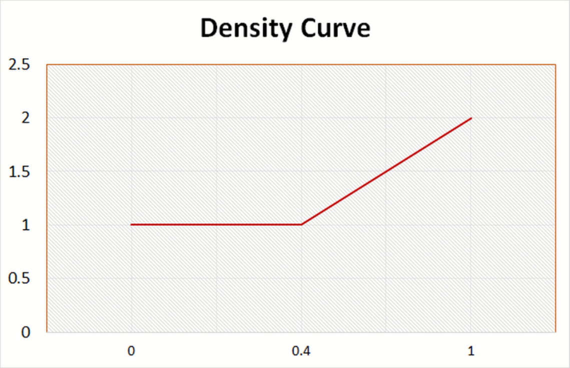


What percent of the observations lie between 0.4 and 0.8?



UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

What percent of observations are equal to 0.4?



Example of a density curve: Uniform distribution

- The **Uniform Distribution** describes a variable that takes values that are uniformly spread between a range of values.
- Thus it takes on a rectangular shape. The proportion (percent) of observations that lie within a range of values is equivalent to the area of the rectangle between the desired range of values.

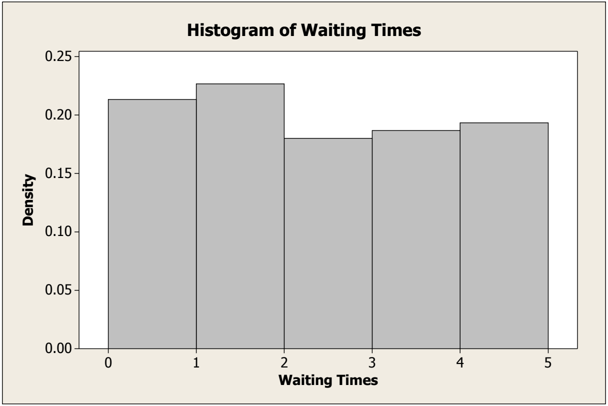
$$Area = Height \times Width$$

The height of the rectangle is $\frac{1}{\text{highest value} - \text{lowest value}}$.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Uniform distribution

The following histogram is of waiting times for an elevator where the longest waiting time is 5 minutes.



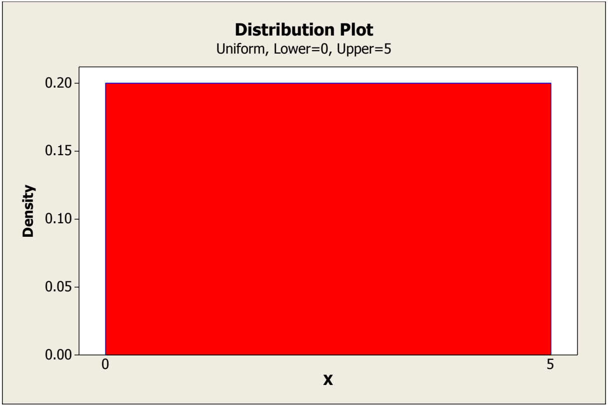
Uniform distribution example

- The random variable X is the waiting time for the elevator.
- The possible values are $0 \leq X \leq 5$.
- Since we are looking at a random variable that assumes values corresponding to an interval this is a **continuous** random variable.
- The probabilities for this random variable is the same as the area under a density curve.
- In this example any one of the times has an equally likely chance of assuming a value between 0 and 5. Thus this curve is rectangular.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Density curve for waiting time

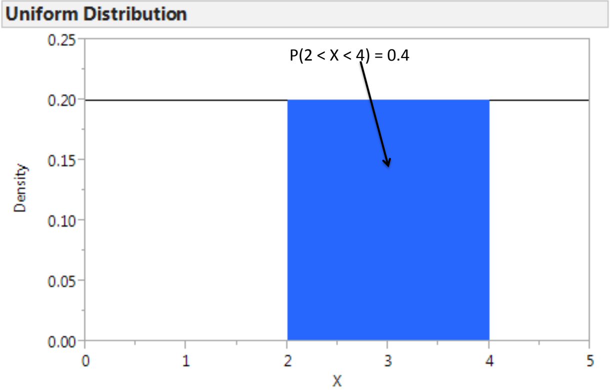
The rectangle ranges between 0 and 5. The height of the rectangle is: $\frac{1}{\text{highest value} - \text{lowest value}} = \frac{1}{5-0} = 0.2.$



$P(2 < X < 4)$

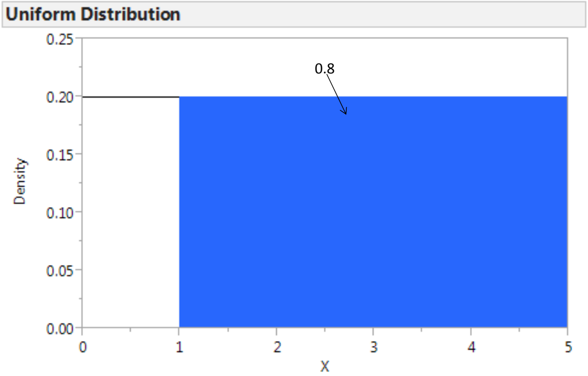
The probability of any event between a range of values is the same as the area between the range under the density curve.

$$\text{Area of rectangle} = \text{height} \times \text{width} = 0.2 \times 2 = 0.4$$



Example continued

What is the probability that a person waits for at least one minute?



Example continued

What is the probability that a person waits for at least one minute?

$$\begin{aligned} P(X \geq 1) &= \text{area above } 1 \\ &= \text{height} \times \text{width} \end{aligned}$$

Example continued

What is the probability that a person waits for at least one minute?

$$\begin{aligned} P(X \geq 1) &= \text{area above } 1 \\ &= \text{height} \times \text{width} \\ &= 0.2 \times 4 \\ &= 0.8 \end{aligned}$$

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Popper Questions

The waiting time for an elevator has a uniform distribution waiting no longer than 5 minutes. Determine the following probabilities using this information.

4. $P(2 \leq X \leq 4)$

   a. 0.4 b. 0.2 c. 0 d. 0.5

5. The probability that a person waits for an elevator for more than 5 minutes.

   a. 0.4 b. 0.2 c. 0 d. 0.5

6. The probability that person waits for an elevator for less than 2.5 minutes.

   a. 0.4 b. 0.2 c. 0 d. 0.5

7. The probability that a person waits for an elevator for exactly 1 minute.

   a. 0.4 b. 0.2 c. 0 d. 0.5

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Mean and standard deviation

- The mean of a random variable that has a uniform distribution is:

$$\mu = \frac{\text{highest value} + \text{lowest value}}{2}$$

- The standard deviation of a random variable that has a uniform distribution is:

$$\sigma = \sqrt{\frac{(\text{highest value} - \text{lowest value})^2}{12}}$$

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Waiting time

- The expected waiting time for this elevator is:

$$\mu = \frac{5+0}{2} = 2.5$$

- The standard deviation for the waiting time for this elevator is:

$$\sigma = \sqrt{\frac{(5-0)^2}{12}} = \sqrt{\frac{25}{12}} = \sqrt{2.08333} = 1.4434$$

- We expect the waiting time to be 2.5 minutes give or take 1.4434 minutes or so.
- What is the median?

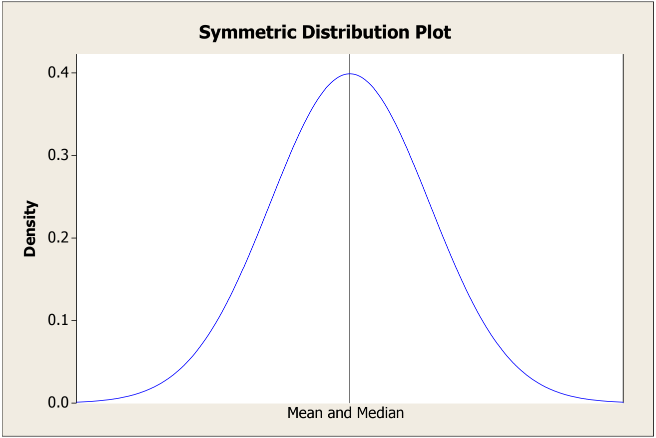
UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

Median and Mean of a Density Curve

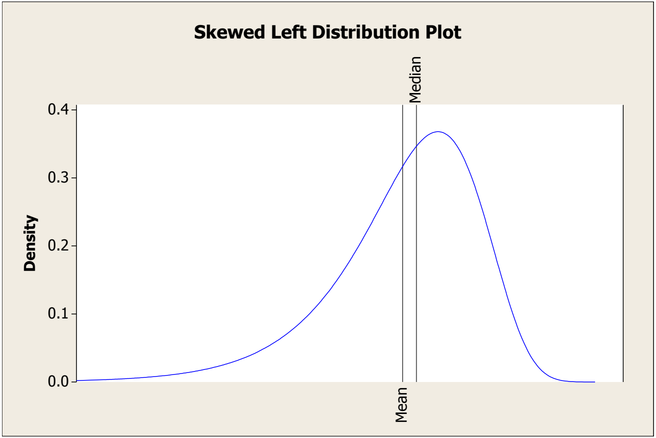
- The median of a density curve is the equal-areas point, the point that divides the area under the curve in half.
- The mean of a density curve is the balance point.
- The mean and the median is the same for symmetric density curve.
- The mean of a skewed curve is pulled away from the median in the direction of the long tail.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS

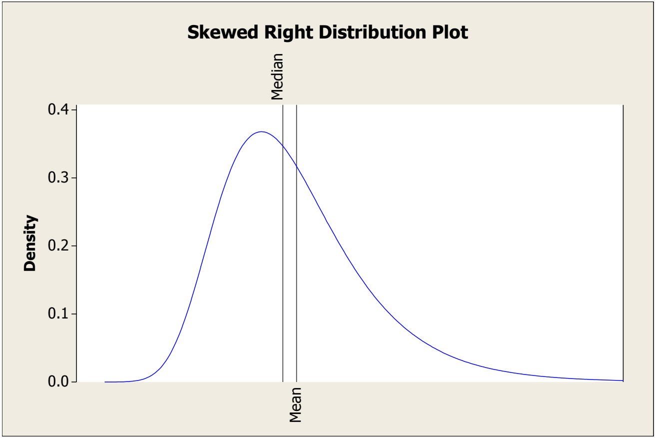
Symmetric Density Curve



Skewed Left Density Curve



Skewed Right Density Curve



Mean and Standard Deviation of a Density Curve

- Density curve is an idealized description of the distribution of data.
- $\bar{x}$ and s are computed from actual observations.
- Mean of an idealized distribution is μ called “mu”.
- Standard deviation of a density curve is σ called “sigma”.

UNIVERSITY of HOUSTON
DEPARTMENT OF MATHEMATICS