Outline

1. Counting Techniques
2. Permutations
3. Combinations
4. Sets
5. Venn Diagrams
Example of Poppers

The following numbers are exams scores from a random sample of 5 students:

65  75  80  85  95

1. Determine the mean
   a) 80  b) 65  c) 95  d) 30

2. Determine the standard deviation
   a) 0  b) 11  c) 80  d) 30
3. This is a standard deviation contest, which list of numbers have the largest standard deviation? No calculations are required.

a) 10, 10, 10, 10
b) 20, 20, 20, 20

c) 10, 10, 20, 20

> sd(c(10,10,20,20))
[1] 5.773503

> sd(c(10,15,15,20))
[1] 4.082483
d) 10, 15, 15, 20
In the city of Milford, applications for zoning changes go through a two-step process:

1. A review by the planning commission.
2. A final decision by the city council.

- At step 1 the planning commission reviews the zoning change request and makes a positive or negative recommendation concerning the change.
- At step 2 the city council reviews the planning commission’s recommendation and then votes to approve or to disapprove the zoning change.

How many possible decisions can be made for a zoning change in Milford?
Counting Rules

- If an experiment can be described as a sequence of $k$ steps with $n_1$ possible outcomes on the first step, $n_2$ possible outcomes on the second step, and so on, then the total number of experimental outcomes is given by $(n_1)(n_2)\ldots(n_k)$.

- A **tree diagram** can be used as a graphical representation in visualizing a multiple-step experiment.

\[ 15 \times 2^4 \]

\[ 2 \times 2 = 4 \]
Tree diagram

Step 1
Planning Commission

positive

negative

Step 2
City Council

approve

disapprove

Sample Points

(positive, approve)

(positive, disapprove)

(negative, approve)

(negative, disapprove)
Examples

- How many ways can you create a pizza choosing a meat and two veggies if you have 3 choices of meats and 4 choices for veggies?

  with replacement  \[3 \times 4 \times 4 = 48\]

  without replacement  \[3 \times 4 \times 3 = 36\]

- In how many ways can 6 people be seated in a row?

  \[6 \times 5 \times 4 \times 3 \times 2 \times 1 = 6! = \text{factorial}(6) = 720\]

- How many possible outcomes can we have when rolling a pair of 6-sided die?

  \[6 \times 6 = 36\]
Permutations

It allows one to compute the number of outcomes when \( r \) objects are to be selected from a set of \( n \) objects where the order of selection is important. The number of permutations is given by

\[
P_r^n = \frac{n!}{(n-r)!} = \text{P}(n, r)
\]

- Where \( n! = n(n-1)(n-2) \cdots (2)(1) \)
- R code for \( n! \): factorial(n)

Seat 6 people in 4 chairs:

\[
P(6, 4) = \frac{6!}{(6-4)!} = \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 1} = 360
\]

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Allowing Repeated Values

When we allow repeated values, The number of orderings of \( n \) objects taken \( r \) at a time, with repetition is \( n^r \).

- Example: In how many ways can you write 4 letters on a tag using each of the letters C O U G A R with repetition?

\[
\begin{align*}
\text{Without repetition:} \\
P(6, 4) &= 360
\end{align*}
\]
Several Objects At Once

The number of permutations, $P$, of $n$ objects taken $n$ at a time with $r$ objects alike, $s$ of another kind alike, and $t$ of another kind alike is

$$P = \frac{n!}{r!s!t!}$$

Example: How many different words (they do not have to be real words) can be formed from the letters in the word MISSISSIPPI?

$$n(M) = 1$$
$$n(I) = 4$$
$$n(S) = 4$$
$$n(P) = 2$$

$$P = \frac{11!}{1!4!4!2!} = \frac{39,916,800}{1152} = 3445$$
Objects Taken of Circular

The number of circular permutations of $n$ objects is $(n - 1)!$.

- Example: In how many ways can 12 people be seated around a circular table?

$\begin{align*}
\quad & n = 12 \\
(12 - 1)! & = 11! = 39,916,800
\end{align*}$
Combinations

Counts the number of experimental outcomes when the experiment involves selecting \( r \) objects from a (usually larger) set of \( n \) objects. The number of combinations of \( n \) objects taken \( r \) unordered at a time is

\[
C_r^n = \binom{n}{r} = \frac{n!}{r!(n-r)!} = \binom{n}{r} = nC_r
\]

Rcode: `choose(n,r)`

\[
C(5,3) = \frac{5!}{3!(5-3)!} = \frac{5 \cdot 4 \cdot 3!}{(3 \cdot 2 \cdot 1)(2 \cdot 1)} = \frac{5 \cdot 2}{2 \cdot 1} = 10
\]

\[
> \text{choose}(5,3)
\]

[1] 10

RStudio
Three Boxes A, B, C selected 2 \( n=3 \) \( r=2 \)

**Permutation**: order matters

\[
P(3,2) = \frac{3!}{(3-2)!} = \frac{3 \cdot 2 \cdot 1}{1} = 6
\]

\( \begin{cases} 
A, B \\
A, C \\
B, A \\
B, C \\
C, A \\
C, B \\
\end{cases} \) \( \text{6 possible permutations} \)

**Combination**

\[
C(3,2) = \frac{3!}{2!(3-2)!} = \frac{3 \cdot 2 \cdot 1}{2 \cdot 1} = 3
\]

\( \begin{cases} 
A, B = B, A \\
A, C = C, A \\
B, C = C, B \\
\end{cases} \) \( \text{3 possible combinations} \)
Examples

- In how many ways can a committee of 5 be chosen from a group of 12 people? \( \binom{12}{5} = \text{choose}(12,5) = 792 \)

- In a manufacturing company they have to choose 5 out of 50 boxes to be sent to a store. How many ways can they choose the 5 boxes? \( \binom{50}{5} = \text{choose}(50,5) = 2,118,760 \)
Examples

1. A researcher selects 3 fish from a tank of 12 and puts each of the 3 fish into different containers. How many ways can this be done?

   $P(12, 3) = 1320$

2. Among 10 electrical components 2 are known not to function. If 5 components are randomly selected, how many ways can we have only one of components not functioning?

   $\binom{2}{1} \times \binom{8}{4} = 140$
Definitions

- A **set** is a collection of objects.
- The items that are in a set called **elements**.
- We typically denote a set by capital letters of the English alphabet.
- Examples: $A = \{\text{knife}, \text{spoon}, \text{fork}\}$, $B = \{2, 4, 6, 8\}$.
- The set $B$ could also be written as $B = \{x | x \text{ are even whole numbers between 0 and 10}\}$.
## Notations of Sets

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a \in A$</td>
<td>The object $a$ is an element of the set $A$.</td>
</tr>
<tr>
<td>$A \subseteq B$</td>
<td>Set $A$ is a subset of set $B$.</td>
</tr>
<tr>
<td></td>
<td>That is every element in $A$ is also in $B$.</td>
</tr>
<tr>
<td>$A \subset B$</td>
<td>Set $A$ is a proper subset of set $B$.</td>
</tr>
<tr>
<td></td>
<td>That is every element that is in $A$ is also in set $B$ and there is at least one element in set $B$ that is not in set $A$.</td>
</tr>
<tr>
<td>$A \cup B$</td>
<td>A set of all elements that are in $A$ or $B$.</td>
</tr>
<tr>
<td>$A \cap B$</td>
<td>A set of all elements that are in $A$ and $B$.</td>
</tr>
<tr>
<td>$U$</td>
<td>Called the universal set, all elements we are interested in.</td>
</tr>
<tr>
<td>$A^C$</td>
<td>The set of all elements that are in the universal set but are not in set $A$.</td>
</tr>
</tbody>
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Examples

The following are sets: \( U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} \), \( A = \{1, 2, 3, 4, 5, 6, 9, 10\} \), \( B = \{3, 4, 7, 8\} \), and \( C = \{2, 3, 9, 10\} \)

- \( C \subset A \)
- \( A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\} = U \)
- \( 9 \in A \) but \( 9 \notin B \)
- \( A^C = \{7, 8\} \)
- \( A \cap B = \{3, 4\} \)
- \( A^C \cap C = \emptyset \)
- \( (B \cup C)^C = \{1, 5, 6\} \)
- \( A \cap B \cap C = \{3\} \)
- \( B \cup C = \{2, 3, 4, 7, 8, 9, 10\} \)
A **Venn diagram** is a very useful tool for showing the relationships between sets.

Venn diagrams consist of a rectangle with one or more shapes (usually circles) inside the rectangle.

The rectangle represents all of the elements that we are interested in for a given situation. This set is the universal set.
Graph of Venn Diagrams

\[ S \]

\[ A \quad A \text{ and } B \quad B \]
Graph of Disjoint Events

mutually exclusive

\[ S \]

\[ A \]

\[ B \]

\[ C \]
$A^C \cap B$
A \cap B \cap C
Soft Drink Preference

A group of 100 people are asked about their preference for soft drinks. The results are as follows: 55 like Coke, 25 like Diet Coke, 45 like Pepsi, 15 like Coke and Diet Coke, 5 like all 3 soft drinks, 25 like Coke and Pepsi, 5 only like Diet Coke (nothing else). Fill in the Venn diagram with these numbers.
Things to Do Before Next Thursday

- Complete quiz 1; closes Wednesday (tomorrow) have 20 attempts. If you are stuck on a problem ask me cathy@math.uh.edu.
- Get the access code for CASA; will not be able to access for free after Saturday.
- Get the poppers; first popper is Thursday.
- Work on homework 1; due on Wednesday (tomorrow).