Math 6320 | Functions of a Real Variable | Fall 2015

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HOMEWORK 1

Due in class *Fri, Aug. 28.*

1. A **relation** on a set X is a subset $R \subset X \times X$. To use notation more compatible with familiar examples of relations such as $=, <, >, \subset, \supset$, write xRy to mean $(x, y) \in R$. An **equivalence relation** is a relation with the following properties:

- **symmetric:** xRy iff yRx
- **transitive:** xRy and yRz implies xRz
- **reflexive:** xRx for every x

Here is a "proof" that every relation R that is both symmetric and transitive is also reflexive: "Since R is symmetric, aRb implies bRa. Since R is transitive, aRb and bRa together imply aRa, as desired." Find the flaw in this argument.

2. Which of the following sets are countable?

(a) The set A of all functions $f\colon \{0,1\} \to \mathbb{N}$.
(b) The set B_n of all functions $f\colon \{1,\dots,n\} \to \mathbb{N}$.
(c) The set $C = \bigcup_{n\in\mathbb{N}} B_n$.
(d) The set D of all functions $f\colon \mathbb{N} \to \mathbb{N}$.
(e) The set E of all functions $f\colon \mathbb{N} \to \{0,1\}$.
(f) The set F of all functions $f\colon \mathbb{N} \to \{0,1\}$ that are "eventually zero". [We say that f is *eventually zero* if there is a positive integer N such that $f(n) = 0$ for all $n \geq N$.]
(g) The set G of all functions $f\colon \mathbb{N} \to \mathbb{N}$ that are eventually 1.
(h) The set H of all functions $f\colon \mathbb{N} \to \mathbb{N}$ that are eventually constant.
(i) The set I of all two-element subsets of $\mathbb{N}$.
(j) The set J of all finite subsets of $\mathbb{N}$.

3. *Folland, 1.2.3.* Let $\mathcal{M}$ be an infinite σ-algebra.

(a) $\mathcal{M}$ contains an infinite sequence of disjoint sets.
(b) $\operatorname{card}(\mathcal{M}) \geq \mathfrak{c}$.

4. If $\mathcal{M}$ is the σ-algebra generated by $\mathcal{E}$, then $\mathcal{M}$ is the union of the σ-algebras generated by $\mathcal{F}$ as $\mathcal{F}$ ranges over all countable subsets of $\mathcal{E}$. (Hint: Show that the latter object is a σ-algebra.)