Math 6320 | Functions of a Real Variable | Fall 2015

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HOMEWORK 11

Due in class *Fri, Nov. 13.*

1. *Folland, 2.64.* For which real values of a and b is $|x|^a \big|\log|x|\big|^b$ integrable over $\{x \in \mathbb{R}^n : |x| < \frac{1}{2}\}$? Over $\{x \in \mathbb{R}^n : |x| > 2\}$?

2. *Folland, 3.2.* If ν is a signed measure, E is ν-null iff $|\nu|(E) = 0$. Also, if ν and μ are signed measures, $\nu \perp \mu$ iff $|\nu| \perp \mu$ iff $\nu^+ \perp \mu$ and $\nu^- \perp \mu$.

3. *Folland, 3.4.* If ν is a signed measure and λ, μ are positive measures such that $\nu = \lambda - \mu$, then $\lambda \geq \nu^+$ and $\mu \geq \nu^-$.

4. *Folland, 3.6.* Suppose $\nu(E) = \int_E f\,d\mu$ where μ is a positive measure and f is an extended μ-integrable function. Describe the Hahn decomposition of ν and the positive, negative, and total variations of ν in terms of f and μ.

5. *Folland, 3.7.* Suppose that ν is a signed measure on $(X, \mathcal{M})$ and $E \in \mathcal{M}$.
(a) $\nu^+(E) = \sup\{\nu(F) : F \in \mathcal{M}, F \subset E\}$ and $\nu^-(E) = -\inf\{\nu(F) : F \in \mathcal{M}, F \subset E\}$.
(b) $|\nu|(E) = \sup\{\sum_{j=1}^n |\nu(E_j)| : n \in \mathbb{N}, E_1, \ldots, E_n$ are disjoint, and $\bigcup_{j=1}^n E_j = E\}$.