HOMEWORK 7

Due in class *Fri, Oct. 9.*

1. *Folland, 2.12.* Prove Proposition 2.20: If $f \in L^+$ and $\int f < \infty$, then $\{x \mid f(x) = \infty\}$ is a null set and $\{x \mid f(x) > 0\}$ is σ-finite. (See Proposition 0.20, where a special case is proved.)

2. *Folland, 2.13.* Suppose $\{f_n\} \subset L^+$, $f_n \to f$ pointwise, and $\int f = \lim \int f_n < \infty$. Then $\int_E f = \lim \int_E f_n$ for all $E \in \mathcal{M}$. However, this need not be true if $\int f = \lim \int f_n = \infty$.

3. *Folland, 2.14.* If $f \in L^+$, let $\lambda(E) = \int_E f \, d\mu$ for $E \in \mathcal{M}$. Then λ is a measure on $\mathcal{M}$, and for any $g \in L^+$, $\int g \, d\lambda = \int fg \, d\mu$. (First suppose that g is simple.)

4. *Folland, 2.17.* Assume Fatou's lemma and deduce the monotone convergence theorem from it.