Math 6320 | Functions of a Real Variable | Fall 2015

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HOMEWORK 9

Due in class *Fri, Oct. 30.*

1. *Folland, 2.32.* Suppose $\mu(X) < \infty$. If f and g are complex-valued measurable functions on X, define

$$\rho(f,g) = \int \frac{|f-g|}{1+|f-g|}\,d\mu.$$

Then ρ is a metric on the space of measurable functions if we identify functions that are equal a.e., and $f_n \to f$ with respect to this metric iff $f_n \to f$ in measure.

2. *Folland, 2.33.* If $f_n \geq 0$ and $f_n \to f$ in measure, then $\int f \leq \liminf \int f_n$.

3. *Folland, 2.34.* Suppose $|f_n| \leq g \in L^1$ and $f_n \to f$ in measure.
(a) $\int f = \lim \int f_n$.
(b) $f_n \to f$ in L^1.

4. *Folland, 2.44.* (Lusin's Theorem). If $f\colon [a,b] \to \mathbb{C}$ is Lebesgue measurable and $\epsilon > 0$, there is a compact set $E \subset [a,b]$ such that $\mu(E^c) < \epsilon$ and $f|_E$ is continuous. (Use Egorov's theorem and Theorem 2.26.)

5. *Folland, 2.46.* Let $X = Y = [0,1]$, $\mathcal{M} = \mathcal{N} = \mathcal{B}_{[0,1]}$, $\mu =$ Lebesgue measure, and $\nu =$ counting measure. If $D = \{(x,x) \mid x \in [0,1]\}$ is the diagonal in $X \times Y$, then $\iint \mathbf{1}_D\,d\mu\,d\nu$, $\iint \mathbf{1}_D\,d\nu\,d\mu$, and $\int \mathbf{1}_D\,d(\mu\times\nu)$ are all unequal. (To compute $\int \mathbf{1}_D\,d(\mu\times\nu) = (\mu\times\nu)(D)$, go back to the definition of $\mu \times \nu$.)