HOMEWORK 4

Due in class *Monday, Mar. 2.*

1. Let $S, \tilde{S}$ be compact surfaces without boundary, and suppose that $\rho\colon \tilde{S} \to S$ is a covering map with degree n. Prove that $\chi(\tilde{S}) = n\chi(S)$.

You may use without proof the following result: *Given any open cover $\mathcal{U}$ of a surface S, there is a triangulation $\mathcal{T}$ of S such that every triangle in $\mathcal{T}$ is completely contained in some element $U \in \mathcal{U}$.*

2. Let S_k be the sphere with k handles. Give a necessary and sufficient condition on $k, \ell \in \mathbb{N}$ for the existence of a covering map $\rho\colon S_k \to S_\ell$.

Hint: Use the previous problem to determine a natural necessary condition. Then given k, ℓ satisfying this condition, describe a particular realization of S_k as a surface in $\mathbb{R}^3$ that is symmetric under rotation by $2\pi/(k-1)$ around the z-axis. Then for a suitable value of n, rotation by $2\pi/n$ around the z-axis will induce an equivalence relation on S_k, whose quotient space is S_ℓ, and whose quotient map is the desired covering map ρ.

3. Consider the function $f(x, y) = \sin(4\pi x)\cos(6\pi y)$ on the torus $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$.

(a) Prove that this is a Morse function (every critical point is non-degenerate) and calculate the number of minima, saddles, and maxima.

(b) Describe the evolution of the sublevel sets $f^{-1}((-\infty), c))$ as c varies from the lowest minimum value to the highest maximum value.