Math 7350 Geometry of Manifolds Spring 2015
Dr. Vaughn Climenhaga, PGH 665

HOMEWORK 5

Due in class *Mon, Mar. 9.*

1. *Lee, Exercise 4-7.* Prove the following statements from Proposition 4.6.
 (f) Every bijective local diffeomorphism is a diffeomorphism.
 (g) A map between smooth manifolds is a local diffeomorphism if and only if in a neighbourhood of each point of its domain, it has a coordinate representation that is a local diffeomorphism.

2. *Lee, Problem 4-4.* Let $\gamma\colon \mathbb{R} \to \mathbb{T}^2$ be the curve given by $\gamma(t) = (e^{2\pi i t}, e^{2\pi i \alpha t})$, where α is any irrational number. Use Lemma 4.21 to show that the image set $\gamma(\mathbb{R})$ is dense in $\mathbb{T}^2$.

3. *Lee, Problem 4-6.* Let M be a nonempty smooth compact manifold. Show that there is no smooth submersion $F\colon M \to \mathbb{R}^k$ for any $k > 0$.

4. *Lee, Problem 4-8.* (This problem shows that the converse of Theorem 4.29 is false.) Let $\pi\colon \mathbb{R}^2 \to \mathbb{R}$ be given by $\pi(x, y) = xy$. Show that π is surjective and smooth, and for each smooth manifold P, a map $F\colon \mathbb{R} \to P$ is smooth if and only if $F \circ \pi$ is smooth; but π is not a smooth submersion.