Math 7350	Geometry of Manifolds	Spring 2015
	Dr. Vaughn Climenhaga, PGH 665	

HOMEWORK 6

Due in class *Fri, Mar. 27.*

1. *Lee, Problem 5-4.* Let $\beta\colon (-\pi, \pi) \to \mathbb{R}^2$ be the smooth curve given by $\beta(t) = (\sin 2t, \sin t)$. (See Example 4.19.) Show that the image of this curve is not an embedded submanifold of $\mathbb{R}^2$. *Be careful: this is not the same as showing that* β *is not an embedding!*

2. *Lee, Problem 5-10.* For each $a \in \mathbb{R}$, let M_a be the subset of $\mathbb{R}^2$ defined by

$$m_a = \{(x, y) \mid y^2 = x(x-1)(x-a)\}.$$

For which values of a is M_a an embedded submanifold of $\mathbb{R}^2$? For which values can M_a be given a topology and smooth structure making it into an immersed submanifold?

3. *Lee, Problem 5-15.* Show by example that an immersed submanifold $S \subset M$ might have more than one topology and smooth structure with respect to which it is an immersed submanifold.

4. *Lee, Problem 5-18(a).* Suppose M is a smooth manifold and $S \subset M$ is a smooth submanifold. Show that S is embedded if and only if every $f \in C^\infty(S)$ has a smooth extension to a neighbourhood of S in M.

See the remark before Lemma 5.34 for the proper definition of $C^\infty(S)$. *Hint: if* S *is not embedded, let* $p \in S$ *be a point that is not in the domain of any slice chart. Let* U *be a neighbourhood of* p *in* S *that is embedded, and consider a function* $f \in C^\infty(S)$ *that is supported in* U *and equal to* 1 *at* p.

5. *Lee, Problem 5-20.* Show by giving a counterexample that the conclusion of Proposition 5.37 may be false if S is merely immersed. That is, give an example of a smooth immersed submanifold $S \subset M$ and $p \in S$ for which T_pS does not coincide with $\{v \in T_pM \mid vf = 0$ whenever $f \in C^\infty(M)$ and $f|_S = 0\}$.