HOMEWORK 2

Due in class *Wed, Feb. 7.*

1. (Bass 12.2–3) Let μ be a signed measure on $(X, \mathcal{A})$ and define integration with respect to μ by $\int f\,d\mu = \int f\,d\mu^+ - \int f\,d\mu^-$.

(a) Prove that
$$\Big| \int f\,d\mu \Big| \le \int |f|\,d|\mu|.$$

(b) Prove that
$$|\mu|(A) = \sup \Big\{ \Big| \int_A f\,d\mu \Big| : |f| \le 1 \Big\}.$$

2. (Bass 12.5) Let $(X, \mathcal{A})$ be a measurable space, and λ a signed measure. Suppose that $\lambda = \mu - \nu$, where μ and ν are finite positive measures. Prove that $\mu(A) \ge \lambda^+(A)$ and $\nu(A) \ge \lambda^-(A)$ for every $A \in \mathcal{A}$.

3. (Bass 12.7) Suppose that μ is a signed measure on $(X, \mathcal{A})$. Prove that if $A \in \mathcal{A}$, then
$$\mu^+(A) = \sup\{\mu(B) : B \in \mathcal{A}, B \subset A\}$$
and
$$\mu^-(A) = -\inf\{\mu(B) : B \in \mathcal{A}, B \subset A\}.$$