HOMEWORK 7

Due in class *Fri, Apr. 27.*

1. (Bass 16.1) Find the Fourier transform of $\mathbf{1}_{[a,b]}$, and in particular, find the Fourier transform of $\mathbf{1}_{[-n,n]}$.

2. (Bass 16.2) Find a real-valued function $f \in L^1$ such that $\hat{f} \notin L^1$.

3. (Bass 16.4) If f is integrable, real-valued, and all the partial derivatives $f_j = \partial f/\partial x_j$ are integrable, prove that the Fourier transform of f_j is given by $\hat{f}_j(u) = -iu_j\hat{f}(u)$.

4. (Bass 16.5) Let $\mathcal{S}$ be the class of real-valued functions f on $\mathbb{R}$ such that for every $k \geq 0$ and $m \geq 0$, we have $|x|^m|f^{(k)}(x)| \to 0$ as $|x| \to \infty$, where $f^{(k)}$ is the kth derivative of f when $k \geq 1$, and $f^{(0)} = f$. The collection $\mathcal{S}$ is called the *Schwartz class*. Prove that if $f \in \mathcal{S}$, then $\hat{f} \in \mathcal{S}$.

5. (Folland 8.16) Let $f_k = \mathbf{1}_{[-1,1]} * \mathbf{1}_{[-k,k]}$.

(a) Compute $f_k(x)$ explicitly and show that $\|f\|_u = 2$, where $\|\cdot\|_u$ is the uniform norm.

(b) Find $g_k \in L^1$ such that $\hat{g}_k = f_k$; show that $\|g_k\|_1 \to \infty$ as $k \to \infty$.

(c) Prove that $\mathcal{F}(L^1)$ is a proper subset of C^0. (Hint: consider g_k and use the open mapping theorem.)