HOMEWORK 4

Due in class *Fri, Mar 23.*

1. (BG 2.7.1). Give an example of a smooth vector field on the sphere with exactly one zero.

2. (BG 2.7.5). Give an example of a flow on the projective plane $\mathbb{R}P^2$ such that there is exactly one fixed point and all other orbits are periodic.

3. (BG 2.7.10). Let M be a smooth m-dimensional manifold, p be a point in M, and $X_1, \ldots, X_k$ be k linearly independent smooth vector fields defined in a neighborhood of p. Show that there exists a local coordinate system $(x_1, \ldots, x_m)$ near p with $(\partial/\partial x_i)|_p = X_i$ for all $1 \leq i \leq k$ if and only if $[X_i, X_j] = 0$ for all $1 \leq i, j \leq k$.

4. (Lee 7.9). Show that the formula $A \cdot [x] = [Ax]$ defines a smooth, transitive left action of $GL(n+1, \mathbb{R})$ on $\mathbb{R}P^n$.

5. (Lee 8.19). Show that $\mathbb{R}^3$ with the cross product is a Lie algebra.

6. (Lee 9.7). Let M be a connected smooth manifold. Show that the group of diffeomorphisms of M acts transitively on M: that is, for any $p, q \in M$, there is a diffeomorphism $F\colon M \to M$ such that $F(p) = q$.

Hint: first prove that if $p, q \in B(0,1) \subset \mathbb{R}^n$, the open unit ball, then there is a compactly supported smooth vector field on $B(0,1)$ whose flow θ satisfies $\theta_1(p) = q$.