# Math 7394, Ergodic theory and thermodynamic formalism, Spring 2019

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**Contact details**
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**Course times:**

- *Lectures:* MWF 11:00–11:50am, AH 2
- *Office hours:* Monday and Wednesday 2–3pm, or by appointment

**Textbook:** The primary text is "An Introduction to Ergodic Theory", by Peter Walters. Another useful textbook is "Foundations of Ergodic Theory", by Marcelo Viana and Krerley Oliveira. I will also refer to "Equilibrium States and the Ergodic Theory of Anosov Diffeomorphisms", by Rufus Bowen, as well as various other primary sources from the research literature.

**Course Description:** Ergodic theory is a central part of the theory of dynamical systems, studying the asymptotic statistical properties of systems evolving in time that preserve an invariant measure. Systems with chaotic behavior generally possess many invariant measures, and thermodynamic formalism borrows tools from statistical mechanics to select a distinguished measure that is physically relevant. The first part of the class will cover topics in classical ergodic theory, including Birkhoff's ergodic theorem, entropy, and the classification of Bernoulli automorphisms.

The remainder of the course will discuss thermodynamic formalism, including the description of Sinai-Ruelle-Bowen measure via absolute continuity, the description of Parry measure via a variational principle, and the connection between the two via the general theory of equilibrium states. Some time will be spent describing the different approaches to thermodynamic formalism and SRB measures in uniform hyperbolicity: Ruelle-Perron-Frobenius operators indirectly via symbolic dynamics or directly via anisotropic Banach spaces; specification and expansivity; and the geometric approach via averaged pushforwards. Time permitting, we will discuss connections to dimension theory and geometric measure theory, and will conclude with a discussion of the nonuniformly hyperbolic setting.

**Class notes:** None of the textbooks listed above contains all of the material that we will cover in this course, and thus in order to have a record of the lectures and of the class we will be producing a set of class notes as we go along. To this end, for each lecture there will be one student designated as the "scribe" for that day, and that student will be responsible for typing up the notes from that lecture (using LaTeX) and sending them to me for inclusion in the main document.

- I will provide a template file with the general LaTeX setup, so that all you need to do is type the material from the lecture itself, rather than worrying about preamble, etc.
- Depending on exactly how many students end up taking the class, I expect each person to be responsible for between four and six lectures.
- I do not expect the notes to be particularly polished or to contain a great deal of "smoothing out" text; the main point is to have a record of exactly what we cover, and to give you practice in using LaTeX, since this is an essential skill for a mathematician.

**Grading:** Your final grade will be determined by the following components (in equal measure).

(1) Attendance and participation – be at the lectures, ask and answer questions when necessary.
(2) Contribution to class notes – see previous item.
(3) Occasional homework assignments.
(4) Final exam/project – likely a take-home exam, but more details to come later.

**General expectations:** This is a "special topics" course, rather than a "prelim preparation course". On the one hand, this means fewer homework assignments and tests, since I am less concerned with evaluating your performance and more concerned with giving you an overview of the field of ergodic theory and thermodynamic formalism. On the other hand, this means that I will at times move rather more quickly than would be appropriate for a prelim-level course; some topics will not be treated in the detail that they really deserve, since this is the only way that I can tell the story I want to tell in the span of a single semester.

My goal is that by the end of the semester, you will have a reasonable bird's-eye view of the main results in ergodic theory and thermodynamic formalism for uniformly hyperbolic systems. Your part in this is to attend lectures regularly, ask questions when something I say is not clear to you, and to think about the material outside of class time: think about answers to informal questions that I pose; think of questions that I might have asked but didn't; talk to your classmates and to me about all of those questions, about the homework, about how all of this fits together.[1]

**Email communications:** When the need arises to contact students individually or as a group, I will do so via the email addresses recorded in the official class list; typically this a "`@uh.edu`" address, unless the student has changed it to something else. I will send a test message to these addresses during the first week of class, and thereafter will assume that any email I send to these addresses has been received by the student. Thus, you are responsible for the content in emails sent to your UH account, regardless if your external (non-UH) email provider filters or blocks them. Emails lost to external providers shall not be used as a justification to claim faculty are unresponsive, to appeal grades, etc.

**Academic honesty and dishonesty:** You are expected to follow the Academic Honesty Policy in the Student Handbook. In particular, the following are expected in this course.

- You are permitted and encouraged to work collaboratively with your classmates on homework assignments to discover and understand solutions – working together and teaching each other is one of the best ways to fully learn the material. However, the final write-up of the solutions must be in your own words.
- Academic dishonesty on exams includes but is not limited to copying work and using prohibited materials such as notes, calculators, or cell phones. Cheating on tests or exams will result in disciplinary action both in this course and at the department and college levels.

**Special needs:** Whenever possible, and in accordance with 504/ADA guidelines, the University of Houston will attempt to provide reasonable academic accommodations to students who request and require them. Please call the Center for Students with DisAbilities (CSD) at 713-743-5400 for more assistance. Do not hesitate to meet with me to discuss such concerns/needs.

**UH CAPS Statement:** Counseling and Psychological Services (CAPS) can help students who are having difficulties managing stress, adjusting to the demands of a professional program, or feeling sad and hopeless. You can reach CAPS (`www.uh.edu/caps`) by calling 713-743-5454 during and after business hours for routine appointments or if you or someone you know is in crisis. No appointment is necessary for the "Let's Talk" program, a drop-in consultation service at convenient locations and hours around campus. `http://www.uh.edu/caps/outreach/lets_talk.html`

[1] Of course I expect any written solution on an assignment or exam to be in your own words, and no one else's.