# FAQ: INVITATION-ONLY CALCULUS, MATH 2413 AND 2414

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## Why is there an "invitation-only" section, and why should I join it?

This section exists to provide a more in-depth and thorough introduction to calculus than in the large-lecture sections or in the AP calculus class you may have taken in high school. In particular, this section emphasizes the role of calculus as a gateway to more advanced and abstract mathematics, and explores some important scientific applications.

*The goal of this section is to tell you the full story of calculus, not just its mechanics.* The large-lecture calculus sections focus on developing the *ability to apply* the techniques of calculus in the *basic settings* where they arise most frequently. To this end, they spend a great deal of time on working examples and explaining the details of how to apply the techniques in different situations. This leaves little time to develop the *theoretical background* that drives these techniques, or to explore some of the more *advanced and in-depth applications*.

This large-lecture approach serves a broad range of students who need to take calculus; for many of these students, calculus is one of their final mathematics courses. However, for students whose future plans involve mathematics more deeply, calculus is not an end, but a beginning, and the goal of this section is to lay a solid foundation on which their future mathematical careers will build. By telling the full story of the definitions, theorems, proofs, and more advanced applications, this section allows these students not only to learn calculus more deeply and completely, but also to gain mathematical maturity and begin to experience what advanced and research-level mathematics feels like. If you plan to take upper-level mathematics courses past calculus, linear algebra, and differential equations, or to pursue a career that involves mathematics and precise reasoning, then this more comprehensive treatment will be beneficial to you.

*Further benefits for math majors.* If you are a math major, or considering becoming one, then it is important for you to meet other math majors and to have direct contact with professors; building community with other students in the major can help support you throughout your undergraduate career, and building good relationships with professors is valuable for opening up research, internship, and job opportunities later on. However, it is difficult to do this in the large lecture calculus classes, for a number of reasons:

- most students in those classes come from other majors, so there may only be a handful of math majors, who you may have trouble finding and meeting properly;
- those classes are taught by instructional faculty, who are excellent teachers but who are not generally research-active, and who are not as well placed to connect you with undergraduate research projects and other opportunities later on;
- the large classes make it difficult for instructors to get to know individual students.

In contrast, this section contains mostly math majors, and is small enough for the instructor to know all the students in the class, which has led to undergraduate research projects, letters of recommendation for summer internships and study abroad programs, etc.

**I have been invited to join the invitation-only section of Math 2413, but I already have AP credit for Calculus and could enroll directly in Math 2414 or 2415. Why should I take this class and repeat the material?**

As suggested in the previous answer, this class will tell you the full story of calculus in a way that you probably did not see it in your AP class. In that class you learned to carry out the mechanics of calculus, such as computation of limits, derivatives, and integrals for specific examples, but it is likely that you did not explore the complete theory that lies behind these tools. You likely know that $(fg)' = f'g + fg'$, but do you know the geometric intepretation that helps explain why the formula is this instead of $f'g'$, or how the "add and subtract" idea in its proof is useful elsewhere too? You may have heard l'Hospital's rule, that $\lim \frac{f}{g} = \lim \frac{f'}{g'}$ under certain assumptions (do you remember them?); but such an assertion needs to be justified! Did you see the proof? You probably learned the Mean Value Theorem, but did you learn why it is true? For math majors especially, the transition to proof-based mathematics is one of the most important aspects of your undergraduate mathematics education. This will begin in earnest when you take Math 3325 and 3333 later on, but it is helpful for you to begin the process now, when the stakes are somewhat lower, and this is an opportunity this section of calculus offers you.

Another part of the story that you probably did not see in AP Calculus has to do with the scientific applications of calculus. You have probably heard about Newton's law of gravitation, and Kepler's laws of planetary motion, but have you seen how calculus can be used to link the two? When you see a power line hanging between two poles, it makes a shape called a *catenary*; have you seen how calculus can be used to describe this shape precisely? In this section we will explore more in-depth applications such as these.

If you are a math major, then another consideration is that *by skipping this class, you would miss out on the opportunity it provides to get to know other math majors and math professors.* In own personal experience, developing good social and professional relationships with other mathematicians was a crucial part of my education, and I learned a great deal from my classmates as well as from my professors. In the large-lecture sections, math majors are in the minority; in this small, invitation-only section, there are more math majors, and you have the opportunity to develop a community of math majors that you can remain a part of throughout your entire undergraduate career.

Regardless of your major, the chance to get to know a research-active professor is also valuable. UH offers various opportunities for undergraduate research, such as PURS (Provost's Undergraduate Research Scholarship) and SURF (Summer Undergraduate Research Fellowship); the professors who teach the invitation-only section can be mentors for these, or can help connect you with other faculty members who can serve as mentors. Similarly, if and when you need letters of recommendation for summer internships, job applications, etc., it is important to have a faculty member who knows you well enough to write a good letter.

Another reason is a little less concrete, but no less important. For many of us who study mathematics, it is a passion, an obsession, and the internal structure of the subject is profoundly beautiful and inspiring. You may have begun to catch glimpses of this beauty; perhaps this is one of the things that draws you to mathematics. In addition to the practicalities above, one of my goals in this section is to give you a chance to encounter some of the ways in which mathematics continues to surprise me, startle me, and bring me joy.

**How is this section different from the regular sections?**

The biggest differences are class size (the invitation-only section is much smaller), class composition (more math majors) and the way that lecture time is allocated. In this section we spend much less time on examples; the expectation is that students will master the concepts quickly enough that fewer examples are needed, and that they are motivated and disciplined enough to spend the necessary amount of time consolidating their knowledge without the level of structure and guidance that is provided by the regular sections. This means that you will be operating with less of a safety net than in the other sections; my goal will be to treat you as mathematicians, not just as calculus students.

With that said, we cover the same general material as the regular sections; the difference is that we give complete justifications for the various theorems and techniques that are described. In Calculus I, we begin with a careful discussion of different sets of numbers ($\mathbb{N}$, $\mathbb{Z}$, $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$) and of the precise mathematical language that will be used throughout the course. We also study proofs of various results, from the limit laws all the way up to the Fundamental Theorem of Calculus. In Calculus II, we describe more involved applications that illustrate the power of calculus. In past years this has included the following.

(1) Use integration by parts to prove the "Wallis product" $\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdots$.
(2) Demonstrate that a hanging cable takes the shape of a catenary, which is given in terms of the hyperbolic cosine function.
(3) Describe how simple population models lead to unpredictable behavior, one of the basic examples of chaos theory.
(4) Explain Bézier curves, which play an important role in graphics and design.
(5) Prove the equivalence of different ways to characterize ellipses and conic sections – algebraic, geometric (foci, cross-sections), physical (gravitation).
(6) Use Newton's laws of motion to derive Kepler's laws of planetary motion – the inverse square law of gravitation explains the observed motion of the solar system.

The class schedule is the same as the regular sections: 3 hours of lecture and 3 hours of lab per week, with lectures given by the professor and labs run by a Ph.D. student TA. The class is still lecture-driven as opposed to discussion-based or other formats, but because the class size is much smaller, it is easier for students and the professor to interact during lectures, and the professor has greater flexibility to respond to student questions and interests.

There are other logistical differences as well: this section does not use CASA or Courseware, and students do not need an access code or popper papers. We follow James Stewart's textbook "Calculus: Early Transcendentals", and there is a second optional textbook, "Calculus" by Michael Spivak, that gives more detail on the theoretical background. Assignments, tests, and exams in the invitation-only section are run on a different schedule from the regular sections and are independent of those sections.

**Is this section more difficult?**

Yes and no. The lectures are certainly given at a more advanced level, are more challenging to follow, and the decrease in examples would undoubtedly create problems for many students. After all, there is a reason that the regular sections are not taught in the same way that the invitation-only section is! On the other hand, the students who really need the

extra examples and extra structure are not the target audience for this section, and as the professor *I make every effort to keep the tests and exams in this section at the same level as those in the regular sections.* In particular, although we spend a significant amount of class time doing theoretical proofs and more advanced applications, which makes the lectures more challenging, I do not include these topics on tests and exams; although the theory and concepts are presented at a more advanced level (with the goal of previewing what more advanced courses have in store), I make every effort to ensure that that this section is graded according to the same standards as in the regular sections.

**Does this section cover the same material?**

Broadly speaking, yes. We do not follow the same schedule, since there are some topics that require more time to discuss when we explore the full theoretical background, and other topics that we cover more quickly by leaving some examples and practice problems to the homework and labs. But by the end of the semester, we will have covered essentially the same list of topics as the regular section. Thus it is possible for students to take the invitation-only class for Calculus I, and the regular section for Calculus II, or vice versa. Calculus III is not covered in the invitation-only class at the present time.

**I am currently taking Math 2413 in the large lecture sections, and am interested in switching to the invitation-only section for Math 2414. Is this a good idea?**

Whether or not you are a math major, if you are currently taking the regular section of calculus and find that you master the techniques quickly, and wish that you could spend more time in the lecture doing some more interesting topics, then you are part of the target audience for this class. Please note, however, that because of limitations of space and class size, enrollment in this section for Math 2414 is restricted to those students who we invite directly, based on their performance in Math 2413, and is on a first-come, first-serve basis; once the class is full, no further students will be admitted.

If you are currently taking the regular section of calculus and are having trouble maintaining a high grade, and think you might perform better in a smaller class, then this is probably *not* the class for you. Although the class size is certainly smaller, this section removes some of the aspects of the regular sections that form somewhat of a safety net, such as popper questions, regular quizzes, and so on. If you find that those aspects routinely challenge your understanding and reveal things you did not internalize correctly at first, allowing you to correct yourself, then you should not take this section, because those safety measures that were essential to your success in Math 2413 will not be present here.

On the other hand, if are performing very well in the regular sections, find that those quizzes are somewhat superfluous and do not reveal any misunderstandings, and you feel that the lectures there do not contain as much interesting new material as you would like, then this may well be the class for you.

**I have another question that wasn't addressed by any of these answers.**

If you have any further questions about the class, please feel free to send me an email at `climenha@math.uh.edu` and I will do my best to answer them, or see my website at `https://www.math.uh.edu/~climenha/math2413.html`