Definition:

A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel.
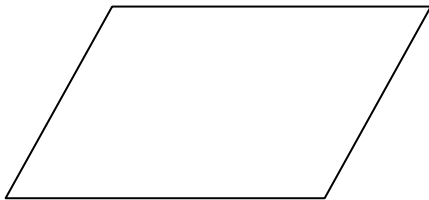
Example 1:

Theorem:

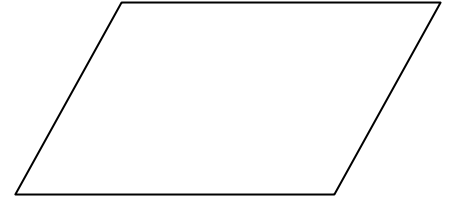A diagonal of a parallelogram separates it into two congruent triangles.

Proof:

Given:
Prove:

Corollary 1: The opposite angles of a parallelogram are congruent.

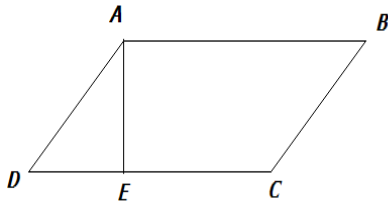**Corollary 2:** The opposite sides of a parallelogram are congruent.

Corollary 3: The diagonals of a parallelogram bisect each other.

Corollary 4: Two consecutive angles of a parallelogram are supplementary.

Definition:

An **altitude** of a parallelogram is a line segment from one vertex that is perpendicular to a non adjacent side (or an extension of that side).
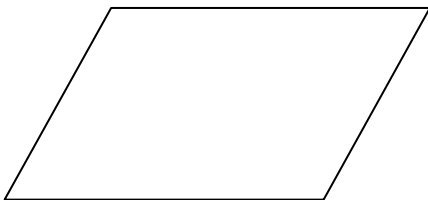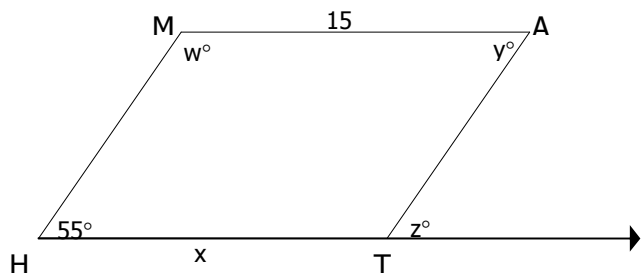
Example 2:

Theorem:

In a parallelogram with unequal pairs of consecutive angles, the longer diagonal lies opposite the obtuse angle.
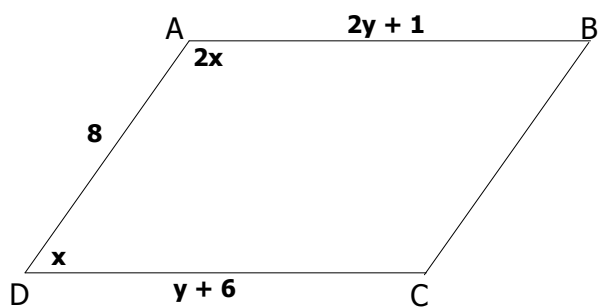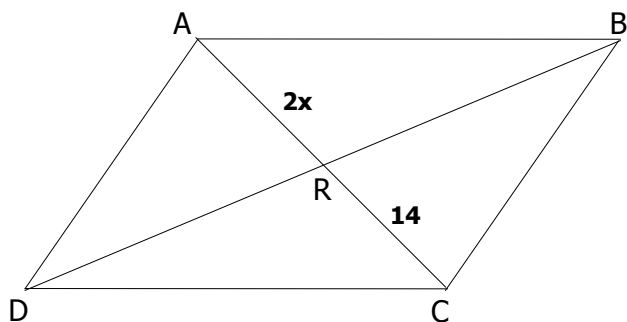
Example 3:

Example 4: MATH is a parallelogram. Find the values of w, x, y, and z.

M ———— 15 ———— A
w°   y°
55°   x
H   T   z°

Example 5: Find the measure of each angle and side in parallelogram ABCD below.

A ———— 2y + 1 ———— B
2x
8
x
D   y + 6   C

Example 6: Diagonals $\overline{AC}$ and $\overline{BD}$ intersect at R. Find x and the measure of $\overline{AC}$.

A   B
2x
R
14
D   C

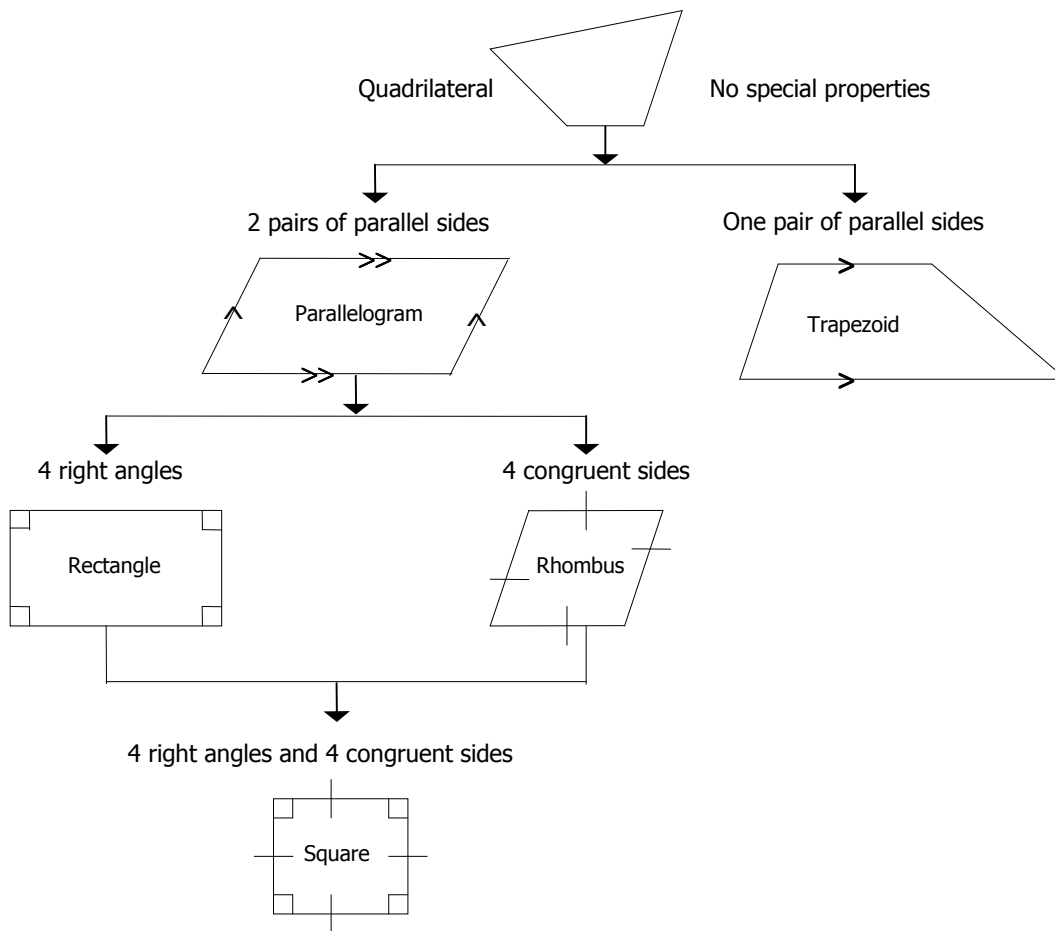**Example 7:** Find the measure of each angle in parallelogram ABCD below.
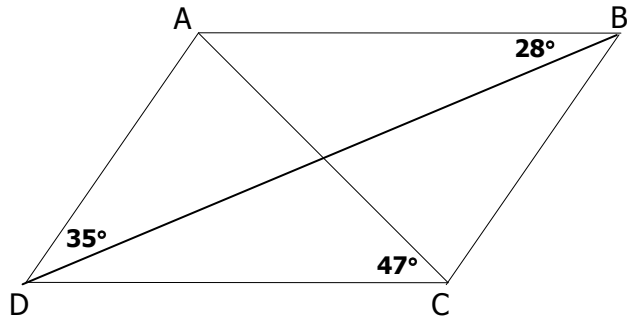
Example 8: Find x and y in the parallelogram below.
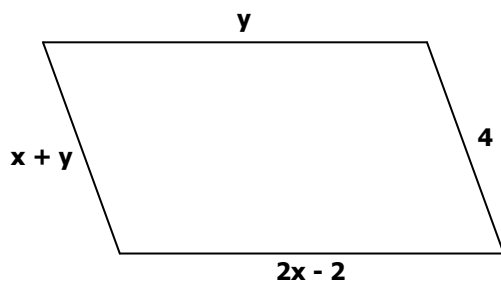
Proving that Quadrilateral is a parallelogram.

As seen in the flow chart below, a rectangle, a rhombus, and a square are all parallelograms.
Hence all "RULES" of parallelograms (listed below) are true for rectangles, rhombi, squares.

"RULES" of parallelograms:

1) Opposite sides of a parallelogram are congruent.
2) Opposite angles of a parallelogram are congruent.
3) Consecutive angles in a parallelogram are supplementary.
4) The diagonals of a parallelogram bisect each other.

Methods might be used to prove that a quadrilateral is a parallelogram.

1. If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.

2. If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

3. If one pair of opposite sides of a quadrilateral are both parallel and congruent, then the quadrilateral is a parallelogram.

4. If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram.

5. If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.

Example 9: If $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$, is quadrilateral ABCD a parallelogram?
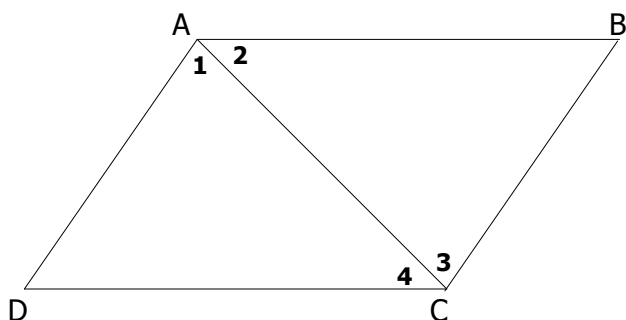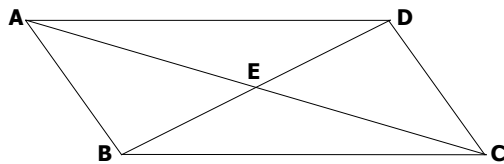

Example 10: State whether or not you can conclude that the figure is a parallelogram, based on the given information.
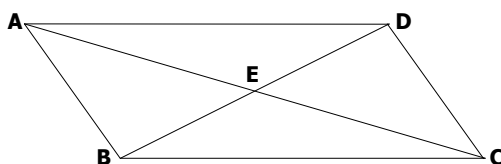
a. $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$

b. $\overline{AB} \parallel \overline{CD}$ and $\overline{AD} \parallel \overline{BC}$

c. $\overline{AB} \cong \overline{CD}$ and $\overline{AB} \parallel \overline{CD}$

Example 10 (continued): State whether or not you can conclude that the figure is a parallelogram, based on the given information.

d. $\overline{AD} \cong \overline{BC}$ and $\overline{AB} \parallel \overline{CD}$

e. $\overline{AE} = \frac{1}{2}\overline{AC}$ and $\overline{BE} = \frac{1}{2}\overline{BD}$

f. $AB = BC = CD = AD$

g. $m\angle ADC = m\angle ABC$ and $m\angle BAD = m\angle BCD$