

Section 4.2 – Annuities: Future Value and Present Value

An **ordinary annuity** is a sequence of equal periodic payments made at the end of each payment period.

Examples of annuities:

1. Regular deposits into a savings account.
2. Monthly home mortgage payments.
3. Payments into a retirement account.

We will study annuities that are subject to the following conditions:

1. The terms are given by fixed time intervals.
2. The periodic payments are equal in size.
3. The payments are made at the end of the payment periods.
4. The payment periods coincide with the interest conversion periods.

The **Future Value of an Annuity** is sum of all payments made and interest earned on an account.

Future Value of an Annuity

The future value F of an annuity of n payments of E dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$F = E \left[\frac{(1+i)^n - 1}{i} \right]$$

F = Future value

E = Equal periodic payment

$$i = \frac{r}{m}$$

r = interest rate

m = compounding periods per year

$$n = mt$$

t = time in years

Example 1: Find the future value of the ordinary annuity of \$1500 per semiannual period for 8 years at 9% per year compounded semiannually.

Present Value of an Annuity

Why would we want to know the present value of an annuity?

Well in some cases we would like to know how much money we should put away now (at the present time), so that at a later time we could withdraw a certain amount of money for a certain length of time. In other cases we may wish to know how much something cost us originally.

The present value P of an annuity of n payments of E dollars each, paid at the end of each investment period into an account that earns interest at the rate of i per period, is

$$P = E \left[\frac{1 - (1 + i)^{-n}}{i} \right]$$

Example 2: Find the present value of the ordinary annuity of \$3000 per quarterly period for 6 years at 11% per year compounded quarterly.

Example 3: Carrie opened an IRA on January 31, 1990, with a contribution of \$2000. She made a contribution of \$2000 thereafter on January 31 of each year until her retirement in the year 2019 (30 payments). If the account earns interest at the rate of 6% per year compounded yearly, how much will Carrie have in her account when she retires?

Example 4: Parents of a college student wish to set up an account that will pay \$350 per month to the student for 4 years. How much should they deposit now at 9% per year compounded monthly, so that their child will have the desired amount of money each month for 4 years?

Example 5: Tim pays \$320 per month for 4 years for a car, making a \$2,000 payment. If the loan borrowed costs 6% per year compounded monthly, what was the original cost of the car?

Example 6: Tom and Jerri paid \$10,000 down toward a new house. They also have a 30-year mortgage for which they pay \$1,100 per month. If interest is 6.35% per year compounded monthly, what did the house that they purchased originally cost?

Example 7: Betsy estimates that she spends \$150 per month on cigarettes. She decides that she will quit and invest the monthly amount spent on cigarettes in an account earning 2.25% per year compounded monthly. At the end of 5 years, she will donate the interest earned on her account to her favorite charity. How much money can the charity expect to get in 5 years?