Section 7.3: Variance and Standard Deviation

The **Variance** of a random variable X is the measure of degree of dispersion, or spread, of a probability distribution about its mean (i.e. how much on average each of the values of X deviates from the mean).

Note: A probability distribution with a small (large) spread about its mean will have a small (large) variance.

Variance of a Random Variable X

Suppose a random variable has the probability distribution

x	x_1	x_2	$\dots$	x_n
$P(X = x)$	p_1	p_2	$\dots$	p_n

and expected value $E(X) = \mu$. Then the variance of the random variable X is

$$Var(X) = p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \cdots + p_n(x_n - \mu)^2$$

Note: We square each since some may be negative.

Standard Deviation measures the same thing as the variance. The standard deviation of a Random Variable X is

$$\sigma = \sqrt{\mathrm{Var}(X)} = \sqrt{p_1(x_1 - \mu)^2 + p_2(x_2 - \mu)^2 + \cdots + p_n(x_n - \mu)^2}$$

Example 1: Compute the mean, variance and standard deviation of the random variable X with probability distribution as follows:

X	*P(X=x)*
-3	0.4
2	0.3
5	0.3

Example 2: An investor is considering two business ventures. The anticipated returns (in thousands of dollars) of each venture are described by the following probability distributions:

Venture 1		Venture 2	
Earnings	Probability	Earning	Probability
-5	0.2	1.5	0.15
30	0.6	50	0.75
60	0.2	100	0.10

a. Compute the mean and variance for each venture.

b. Which investment would provide the investor with the higher expected return (the greater mean)?

c. Which investment would the element of risk be less (that is, which probability distribution has the smaller variance)?

Chebychev's Inequality

Let X be a random variable with expected value μ and standard deviation σ. Then, the probability that a randomly chosen outcome of the experiment lies between $\mu - k\sigma$ and $\mu + k\sigma$ is at least $1 - \frac{1}{k^2}$; that is,
$P(\mu - k\sigma \leq X \leq \mu + k\sigma) \geq 1 - \frac{1}{k^2}$

Example 3: A probability distribution has a mean 20 and a standard deviation of 3. Use Chebychev's Inequality to estimate the probability that an outcome of the experiment lies between 12 and 28.

Example 4: A light bulb has an expected life of 200 hours and a standard deviation of 2 hours. Use Chebychev's Inequality to estimate the probability that one of these light bulbs will last between 190 and 210 hours?