If you find any errors, please let me know.

1432 Test 2 Review

1. Determine if the following are one-to-one, if so, find $f^{-1}(x)$

   a. $f(x) = x^3 + 1$

   b. $f(x) = 3x + 10$

   c. $f(x) = \sqrt{9 - x^2}$

a) $f(x) = x^3 + 1$

$f'(x) = 3x^2 \geq 0 \therefore$ invertible

$y = x^3 + 1$

$y - 1 = x^3$

$\sqrt[3]{y-1} = x$

$\sqrt[3]{x-1} = y = f^{-1}(x)$

b) $f(x) = 3x + 10$

$f'(x) = 3 > 0 \therefore$ invertible

$y = 3x + 10$

$\frac{y-10}{3} = x$

$\frac{x-10}{3} = y = f^{-1}(x)$

c) $f(x) = \sqrt{9 - x^2}$

$f'(x) = \frac{1}{2}(9 - x^2)^{-1/2}(-2x)$

$= \frac{-x}{\sqrt{9 - x^2}}$ positive & negative; therefore not invertible

2. Suppose f has an inverse, $f(3) = 1$ and $f'(3) = \frac{2}{7}$. Find $(f^{-1})'(1)$.

$(f^{-1})'(1) = \frac{1}{f'(3)} = \frac{1}{2/7} = \frac{7}{2}$

3. Suppose *f(x)* is an invertible differentiable function and the graph of *f* passes through the points (6, -1) and (-1, 2). The slope of the tangent line to the graph of *f* at *x* = -1 is 7/2. Find the equation of the tangent line to the inverse of *f* at 2.

$f'(-1) = 7/2 \quad f^{-1}(2) = -1 \quad (f^{-1})'(2) = \frac{1}{f'(-1)} = 2/7$

$(2,-1) \quad m = 2/7 \qquad y + 1 = 2/7(x-2)$

4. Find $(f^{-1})'(a)$ if $f(x) = x^3 + 1$ and $a = 9$

$9 = x^3 + 1$

$x = 2$

$(f^{-1})'(9) = \frac{1}{f'(2)} = \frac{1}{12}$

$f(x) = x^3 + 1$

$f'(x) = 3x^2$

5. a. $y = \ln\sqrt{e^x + 4x} = \frac{1}{2}\ln(e^x + 4x) \qquad y' = \frac{e^x + 4}{2(e^x + 4x)}$

b. $y = \sin\left(\ln(5-x)^6\right)$

$= \sin(6\ln(5-x)) \quad y' = \cos(\ln(5-x)^6) \cdot \frac{-6}{5-x}$

c. $y = x^2e^{2x} + \ln e^{2x} \qquad y' = x^2 \cdot 2e^{2x} + 2xe^{2x} + 2$

d. $y = e^{x^2} \cdot \cosh(3x) \qquad y' = e^{x^2} \cdot 3\sinh(3x) + 2xe^{x^2}\cosh(3x)$

e. $f(x) = \ln\left(\sec\sqrt{x}\right) \qquad y' = \frac{\sec\sqrt{x}\tan\sqrt{x}}{\sec\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} = \frac{\tan\sqrt{x}}{2\sqrt{x}}$

f. $f(x) = \frac{e^{\sqrt{x}}}{x^3} \qquad f'(x) = \frac{x^3 \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}} - e^{\sqrt{x}} \cdot 3x^2}{x^6}$

$$= \frac{\frac{xe^{\sqrt{x}}}{2\sqrt{x}} - 3e^{\sqrt{x}}}{x^4}$$

$$= \frac{e^{\sqrt{x}}(x - 6\sqrt{x})}{2\sqrt{x} \cdot x^4}$$

g. $y = (\cos x)^{(x+7)}$

$$\ln y = (x+7)\ln(\cos x)$$

$$\frac{y'}{y} = (x+7)\cdot\frac{-\sin x}{\cos x} + \ln(\cos x)$$

$$y' = (\cos x)^{(x+7)}\left(-(\tan x)(x+7) + \ln(\cos x)\right)$$

h. $f(x) = (3x-1)^{2x+6}$

$$\ln y = (2x+6)\ln(3x-1)$$

$$y'/y = (2x+6)\cdot\frac{3}{3x-1} + 2\ln(3x-1)$$

$$y' = (3x-1)^{(2x+6)}\left(\frac{3(2x+6)}{3x-1} + 2\ln(3x-1)\right)$$

i. $f(x) = \ln(5x^2) + e^{6x} + \arctan(5-2x)$

$$f'(x) = \frac{10x}{5x^2} + 6e^{6x} + \frac{(-2)}{1+(5-2x)^2}$$

$$= \frac{2}{x} + 6e^{6x} - \frac{2}{1+(5-2x)^2}$$

j. $f(x) = \log_7(3x^2)$ $= \dfrac{\ln 3x^2}{\ln 7}$

$$f'(x) = \frac{1}{\ln 7}\left(\frac{6x}{3x^2}\right) = \frac{2}{x\ln 7}$$

k. $y = 6^{-2x}$ $\quad y' = 6^{-2x}\cdot \ln 6 \cdot -2$

l. $f(x) = \arctan(2x^3)$ $\quad f'(x) = \dfrac{6x^2}{1+4x^6}$

6.

a. $\int_e^{4e} \frac{1}{x}\,dx \quad = \ln|x| \Big|_e^{4e} = \ln 4e - \ln e = \ln \frac{4e}{e} = \ln 4$

b. $\int \left(\frac{\csc^2 x}{2+5\cot x} - e^{9x} \right) dx \quad = \frac{-1}{5}\int \frac{-5\csc^2 x}{2+5\cot x}\,dx - \int e^{9x}\,dx$

$= \frac{-\ln|2+5\cot x|}{5} - \frac{e^{9x}}{9} + C$

c. $\int \frac{\sinh x}{(2+\cosh x)^2}\,dx \quad = \int (2+\cosh x)^{-2} \cdot \sinh x\,dx$

$= \frac{(2+\cosh x)^{-1}}{-1} + C$

$= \frac{-1}{2+\cosh x} + C$

d. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx \quad = 2\int \frac{e^{\sqrt{x}}}{2\sqrt{x}}\,dx = 2e^{\sqrt{x}} + C$

e. $\int \frac{2}{\sqrt{x}\left(3-\sqrt{x}\right)}\,dx \quad = 2\int \frac{\frac{1}{\sqrt{x}}}{(3-\sqrt{x})}\,dx = -2 \cdot 2\int \frac{\frac{-1}{2\sqrt{x}}}{(3-\sqrt{x})}\,dx$

$= -4\ln|3-\sqrt{x}| + C$

f. $\int \frac{x+2}{x+1}\,dx \quad = \int \left(\frac{x+1}{x+1} + \frac{1}{x+1} \right) dx = x + \ln|x+1| + C$

g. $\int \frac{3x^2+3x+3}{x^2+1}dx = \int \frac{3(x^2+1)}{x^2+1} + \frac{3x}{x^2+1}\,dx$

$= 3x + \frac{3}{2}\int \frac{2x}{x^2+1}\,dx$

$= 3x + \frac{3}{2}\ln(x^2+1) + C$

h. $\int \frac{\cos^3 x - \sin^2 x}{\cos^2 x}dx = \int (\cos x - \tan^2 x)\,dx$

$= \sin x - \int (\sec^2 x - 1)\,dx$

$= \sin x - \tan x + x + C$

i. $\int \tan(3x)dx = \frac{1}{3}\int 3\tan 3x\,dx$

$= -\frac{1}{3}\ln|\cos 3x| + C$

$= \frac{1}{3}\ln|\sec 3x| + C$

j. $\int \frac{\arctan(3x)}{1+9x^2}dx = \frac{1}{3}\int (\arctan 3x)\cdot\frac{3}{1+9x^2}\,dx$

$= \frac{1}{6}(\arctan 3x)^2 + C$

k. $\int_0^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1-x^2}}dx = \arcsin x \Big|_0^{\sqrt{3}/2} = \arcsin \sqrt{3}/2 - \arcsin 0$

$= \pi/3$

l. $\int \cos^4 x \sin^3 x dx$ $= \int \cos^4 x \sin^2 x \sin x \, dx$

$$= \int \cos^4 x (1 - \cos^2 x) \sin x \, dx$$

$$= \int (\cos^4 x - \cos^6 x) \sin x \, dx$$

$$= -\frac{\cos^5 x}{5} + \frac{\cos^7 x}{7} + C$$

m. $\int \cos^5 x \sin^2 x dx$ $= \int \cos^4 x \sin^2 x \cos x \, dx$

$$= \int (1 - \sin^2 x)^2 \sin^2 x \cos x \, dx$$

$$= \int (1 - 2\sin^2 x + \sin^4 x) \sin^2 x \cos x \, dx$$

$$= \int (\sin^2 x - 2\sin^4 x + \sin^6 x) \cos x \, dx$$

$$= \frac{\sin^3 x}{3} - \frac{2\sin^5 x}{5} + \frac{\sin^7 x}{7} + C$$

n. $\int \cot^3 x dx$ $= \int \cot^2 x \cot x \, dx$

$$= \int (\csc^2 x - 1) \cot x \, dx$$

$$= \frac{-\cot^2 x}{2} - \ln|\sin x| + C$$

o. $\int x \ln(2x) dx$ $= \frac{x^2}{2} \ln 2x - \int \frac{1}{x} \frac{x^2}{2} dx$

$u = \ln 2x \quad dv = x\,dx$

$du = \frac{1}{x} dx \quad v = \frac{x^2}{2}$

$$= \frac{x^2}{2} \ln 2x - \frac{x^2}{4} + C$$

p. $\int 2x\sin(3x)dx$ $= -\frac{2x\cos 3x}{3} + \frac{2\sin 3x}{9} + C$

$\frac{d}{dx}$	$\int$	
$2x$	$\sin 3x$	$+$
2	$\frac{-\cos 3x}{3}$	$-$
	$\frac{-\sin 3x}{9}$	$+$
		$-$

OR $u = 2x$ $dv = \sin 3x$
$du = 2dx$ $v = \frac{-\cos 3x}{3}$

$$\int 2x\sin 3x\,dx = -\frac{2x\cos 3x}{3} - \int -\frac{\cos 3x}{3}\cdot 2dx$$

$$= -\frac{2x\cos 3x}{3} + \frac{2\sin 3x}{9} + C$$

Q, R, S, T on next test

u. $\int \frac{5}{36+(x-1)^2}dx$ $= 5\cdot\frac{1}{6}\arctan\frac{x-1}{6} + C$

$a = 6$
$u = x-1$
$du = dx$

v. $\int \tan^4(x)dx$ $= \int \tan^2 x\tan^2 x\,dx$

$$= \int(\sec^2 x - 1)\tan^2 x\,dx$$

$$= \frac{\tan^3 x}{3} - \int(\sec^2 x - 1)dx = \frac{\tan^3 x}{3} - \tan x + x + C$$

w. $\int 2x\sec\left(4x^2\right)dx$ $= \frac{1}{4}\int 8x\sec(4x^2)dx$

$u = 4x^2$ $= \frac{1}{4}\ln\left|\sec 4x^2 + \tan 4x^2\right| + c$

$du = 8x\,dx$

x. $\int \sec^4(x)dx$ $= \int \sec^2 x \sec^2 x\,dx$

$= \int (\tan^2 x + 1)\sec^2 x\,dx$

$= \frac{\tan^3 x}{3} + \tan x + C$

7. **Give the general solution for $\frac{dy}{dx} = (y+5)(x+2)$**

$$\frac{dy}{dx} = (y+5)(x+2)$$

$$\int \frac{dy}{y+5} = \int (x+2)dx$$

$$\ln|y+5| = \frac{x^2}{2} + 2x + c$$

$$e^{\frac{x^2}{2}+2x+c} = y+5$$

$$y = Ce^{\frac{x^2}{2}+2x} - 5$$

8. **Find the specific solution given the initial condition: $\frac{dy}{dx} = y - 2 \quad y(0) = 6$**

$\int \frac{dy}{y-2} = \int dx$ $Ce^x = y-2$ $4e^x + 2 = y$

$\ln|y-2| = x + c$ $Ce^0 = 6-2$

$C = 4$

9. The number N of bacteria in a culture is given by $N = 200e^{kt}$. If N = 300 when t = 4 hours, find k (to the nearest tenth) and then determine approximately how long it will take for the number of bacteria to triple in size.

$$300 = 200e^{4k}$$
$$3/2 = e^{4k}$$
$$\ln(3/2) = 4k$$
$$\frac{\ln(3/2)}{4} = k$$

$$\text{TRIPLING TIME} = \frac{\ln 3}{k} = \frac{\ln 3}{\ln(3/2)} \cdot 4$$

10. Suppose that the population of Zeegers grows at a rate proportional to itself, doubling every 12500 years. When the Zeeger population has reached 93 percent more than their current population, they plan to invade Earth. How many years will it be before the Zeegers attack Earth?

$$DT = 12500 \Rightarrow k = \frac{\ln 2}{12500}$$
$$193 = 100e^{kt}$$
$$\ln 1.93 = \frac{\ln 2}{12500} t$$
$$t = \frac{12500 \ln 1.93}{\ln 2} \text{ years}$$

11. At what rate *r* of continuous compounding does a sum of money double in 15 years?

$$DT = 15 \text{ YRS} \Rightarrow r = \frac{\ln 2}{15}$$

12. Give the equation for the tangent and normal to the curve: $f(x) = \ln(2x-5) + e^{x-3}$ at the point (3,1).

$$f'(x) = \frac{2}{2x-5} + e^{x-3}$$
$$f'(3) = 2 + 1 = 3$$

TL: $y - 1 = 3(x-3)$

NL: $y - 1 = -\frac{1}{3}(x-3)$