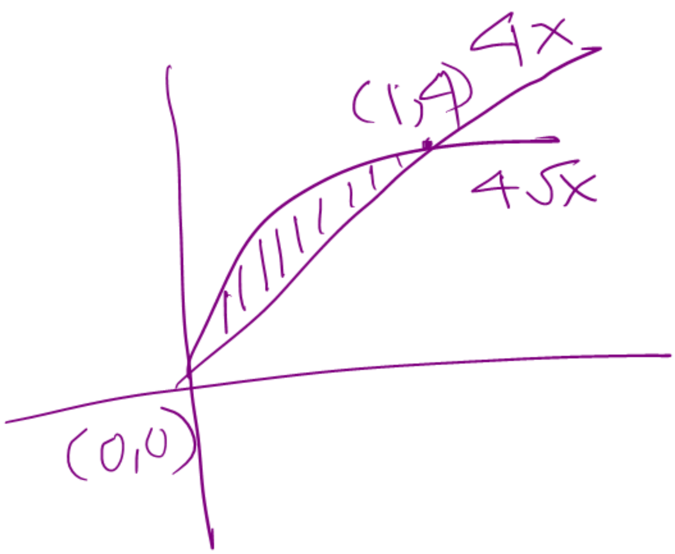
Welcome Math 1432

$f(x) = \sin x \quad g(x) = \cos x$

$\left[0, \frac{\pi}{3}\right]$

Area between $y_1 = 4\sqrt{x}$

$y_2 = 4x$



$$4\sqrt{x} = 4x$$
$$\sqrt{x} = x$$
$$x = x^2$$
$$x^2 - x = 0$$
$$x(x-1) = 0$$
$$x = 0, 1$$

$$\int_0^1 (4\sqrt{x} - 4x)\,dx$$

$$\int_0^1 4\sqrt{x}\,dx - \int_0^1 4x\,dx$$

$$= 4\frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1}\Big|_0^1 - 4\frac{x^{1+1}}{1+1}\Big|_0^1 = \left(\frac{\cancel{4}^{2}}{\cancel{3}}\cdot\frac{2}{3}x^{3/2} - \frac{\cancel{4}^{2}x^2}{\cancel{2}}\right)\Big|_0^1$$

$$= 6 - 2$$
$$= 4$$

For an even function $f(x)$ with an average value of 4 on the interval $[-3, 3]$, determine $\int_0^3 f(x)\,dx$.

a) -24
b) 24
c) 12
d) -12
e) 6

$$\int_0^3 f(x)\,dx$$

$$\text{Avg. value} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$a = -3 \quad b = 3$

$$4 = \frac{1}{3-(-3)}\int_{-3}^{3} f(x)\,dx$$

$$4 = \frac{1}{6}\int_{-3}^{3} f(x)\,dx$$

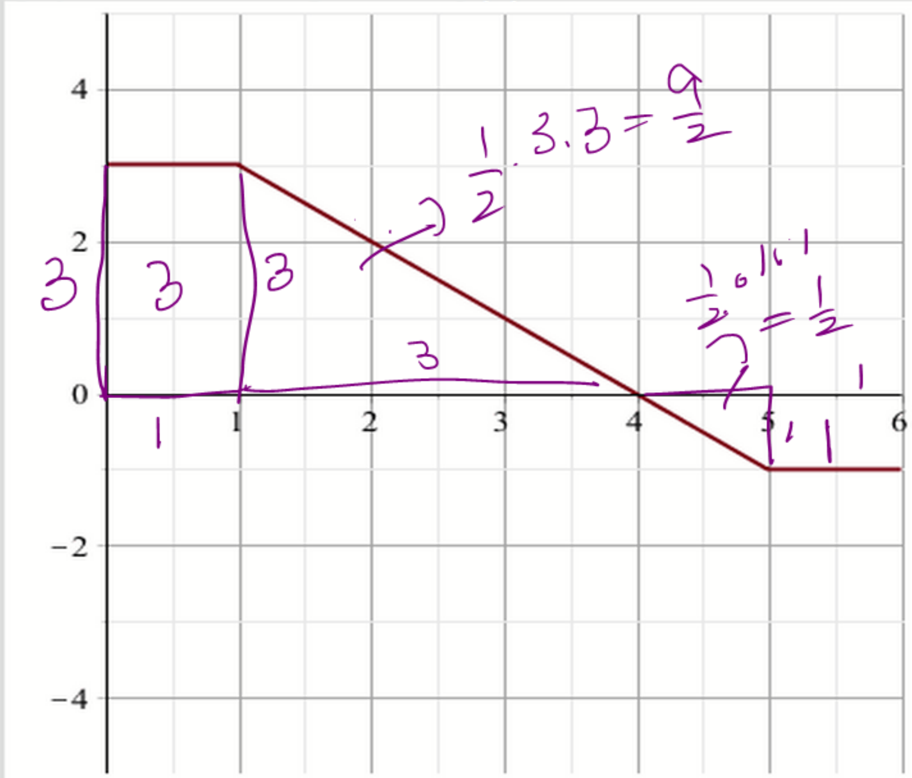
$$24 = \int_{-3}^{3} f(x)\,dx$$

$$24 = \int_{-3}^{0} f(x) + \int_{0}^{3} f(x)\,dx$$

$$24 = 2\int_0^3 f(x)\,dx \qquad \int_0^3 f(x)\,dx = \int_{-3}^{0} f(x)\,dx$$

$$12 = \int_0^3 f(x)\,dx$$

The graph of $f(x)$ is given below. Use this graph to find the average value of $f(x)$ **from** $x = 0$ **to** $x = 6$:



$3 + \frac{9}{2} - \frac{1}{2} - 1$

$2 + 4 = 6$

$\int_0^6 f(x)\,dx = 6$

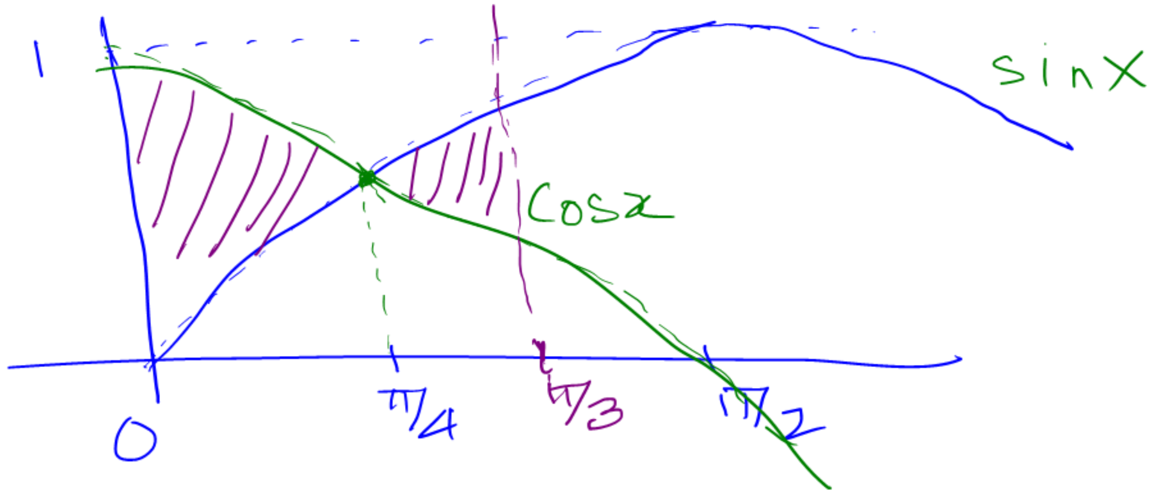
$\text{Avg value} = \frac{6}{6-0}$

$= \frac{6}{6}$

$= 1$

a) ○ $\frac{3}{2}$
b) ○ 9
c) ○ 1
d) ○ 6
e) ○ -1

$f(x) = \sin x \qquad g(x) = \cos x \qquad [0, \frac{\pi}{3}]$



$$\int_0^{\pi/4} (\cos x - \sin x)\,dx + \int_{\pi/4}^{\pi/3} (\sin x - \cos x)\,dx$$

$$= \sin x + \cos x \Big|_0^{\pi/4} + (-\cos x - \sin x)\Big|_{\pi/4}^{\pi/3}$$

$$= \sin x + \cos x \Big|_0^{\pi/4} - (\cos x + \sin x)\Big|_{\pi/4}^{\pi/3}$$

$$= \left\{\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) - (1)\right\} - \left\{\left(\frac{1}{2} + \frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)\right\}$$

$$= \left\{\sqrt{2} - 1\right\} - \left\{\frac{1}{2} + \frac{\sqrt{3}}{2} - \sqrt{2}\right\}$$

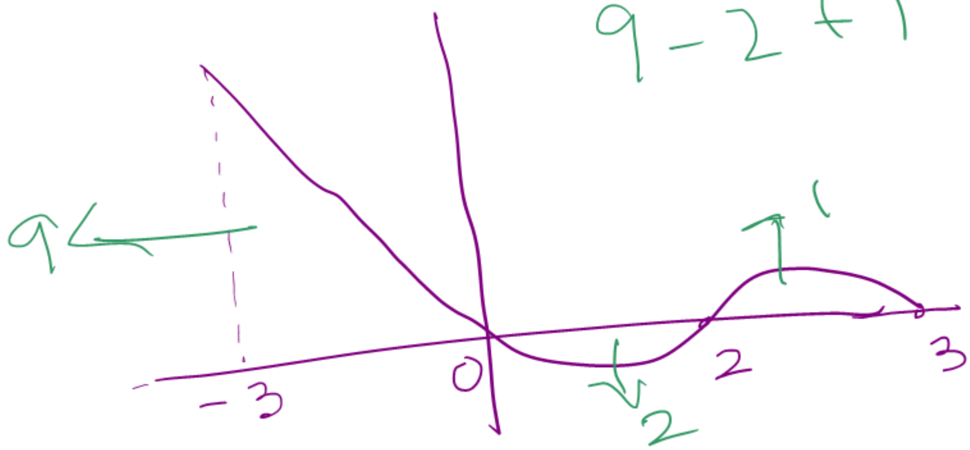
$$= \sqrt{2} - 1 - \frac{1}{2} - \frac{\sqrt{3}}{2} + \sqrt{2}$$

$$= 2\sqrt{2} - \frac{3}{2} - \frac{\sqrt{3}}{2} = 2\sqrt{2} - \frac{3(1+\sqrt{3})}{2}$$

Lab Quiz #1

1. Let the graph of $f(x)$ be as follows:

$9 - 2 + 1 = 8$



The area of $f(x)$ bounded by $[-3,0]$ is 9
" " " " " " $[0,2]$ is 2
" " " " " " $[2,3]$ is 1

What is the average value of $f(x)$ on $[-3,3]$?

(a) $\frac{1}{2}$ (b) $\frac{4}{3}$ (c) 1 (d) $\frac{5}{3}$

$$\text{Avg value} = \frac{1}{3-(-3)} \int_{-3}^{3} f(x)\,dx$$

$$= \frac{1}{6} 8 = \frac{4}{3}$$

$$\int \frac{x+2}{25+49x^2}\,dx$$

$$= \int \frac{x}{25+49x^2}\,dx + \frac{1}{49}\int \frac{2}{\frac{25}{49}+x^2}\,dx$$

$u = 25+49x^2$

$du = 98x\,dx$

$\frac{du}{98} = x\,dx$

$$= \frac{1}{98}\int \frac{du}{u} + \frac{2}{49}\int \frac{2}{\left(\frac{5}{7}\right)^2+x^2}\,dx$$

$$= \frac{1}{98}\ln|u| + \frac{2}{7\cancel{49}}\,\frac{1}{\frac{5}{\cancel{7}}}\tan^{-1}\frac{x}{5/7} + C$$

$$\int \frac{dx}{a^2+x^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$$

$$= \frac{1}{98}\ln|25+9x^2| + \frac{2}{35}\tan^{-1}\frac{7x}{5} + C$$

Which of the following substitutions are correct? (Zero or more options can be correct)

Q2.

- ☐ a) $\int te^{t^2+4}\,dt = \int e^w\,dw$ where $w = t^2+4$.
- ☐ b) $\int \frac{3x^2+4x+1}{x^3+2x^2+x-3}\,dx = \int \frac{1}{w}\,dw$ where $w = x^3+2x^2+x-3$.
- ☐ c) $\int \frac{\cos\sqrt{x}}{2\sqrt{x}}\,dx = \int \cos w\,dw$ where $w = \sqrt{x}$.
- ☐ d) $\int \frac{1}{x^2+4}\,dx = \int \frac{1}{w}\,dw$ where $w = x^2+4$.
- ☐ e) $\int (z^2+2z)(z^3+3z^2-4)^4\,dz = \int \frac{w^4}{3}\,dw$ where $w = z^3+3z^2-4$.

Q3. $\int \frac{e^{\frac{1}{x}}}{x^2}\,dx$

(a) $e^{1/x}+C$ (b) $e^{1/x^2}+C$

(c) $-e^{1/x}+C$ (d) $\frac{3e^{1/x}}{x^3}+C$

(e) $-\frac{3}{2}\frac{e^{1/x^2}}{x^3}+C$

$$\int \frac{\arcsin 9x}{\sqrt{1-81x^2}}\,dx$$

$u = \sin^{-1} 9x$

$du = \dfrac{1}{\sqrt{1-(9x)^2}} \cdot 9\,dx$

$\dfrac{du}{9} = \dfrac{dx}{\sqrt{1-81x^2}}$

$= \frac{1}{9}\int u\,du$

$= \frac{1}{9}\frac{u^2}{2} + C$

$= \dfrac{(\sin^{-1} 9x)^2}{18} + C$

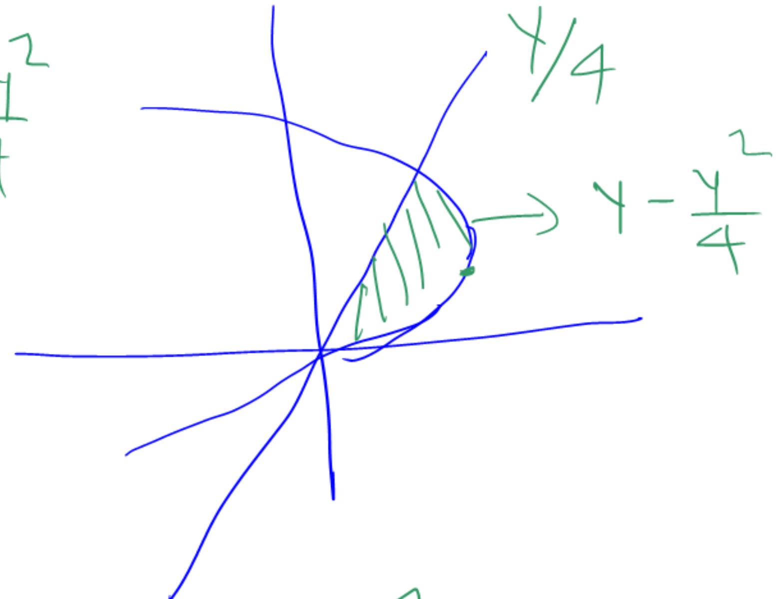
$$\int \frac{t}{\sqrt{1-t^4}}\,dt$$

(*) $4x = 4y - y^2$ $\quad 4x - y = 0$

Area bounded

$$x = \frac{4y - y^2}{4} = y - \frac{y^2}{4}$$

$$x = y/4$$



$$y - \frac{y^2}{4} = \frac{y}{4}$$

$$y - \frac{y}{4} - \frac{y^2}{4} = 0$$

$$\Rightarrow \frac{3y}{4} - \frac{y^2}{4} = 0$$

$$\Rightarrow 3y - y^2 = 0$$

$$\Rightarrow y(3 - y) = 0$$

$$\Rightarrow y = 0, 3$$

$$\int_0^3 \left(\left(y - \frac{y^2}{4}\right) - \frac{y}{4}\right) dy$$

Q4. Find the area of the region enclosed by the graphs $f(x) = \sin x$ & $g(x) = \cos x$ on $[0, \frac{\pi}{2}]$

(a) $1+2\sqrt{2}$ (b) $2(1+\sqrt{2})$ (c) 2

(d) $2(\sqrt{2}-1)$

Q5 Which of the following represents the area of the region bounded by the curves $f(x) = \sqrt{x-1}$, $g(x) = \frac{2}{x}$, $y=0$ and $x=4$.

(a) $\int_0^4 \left(\sqrt{x-1} + \frac{2}{x}\right) dx$

(b) $\int_1^2 \left(\frac{2}{x} - \sqrt{x-1}\right) dx + \int_2^4 \left(\sqrt{x-1} - \frac{2}{x}\right) dx$

(c) $\int_1^2 \sqrt{x-1}\, dx + \int_2^4 \frac{2}{x}\, dx$

(d) $\int_0^2 \sqrt{x-1}\, dx + \int_2^4 \left(\sqrt{x-1} - \frac{2}{x}\right) dx$

(e) $\int_0^2 \sqrt{x-1}\, dx + \int_2^4 \frac{2}{x}\, dx$