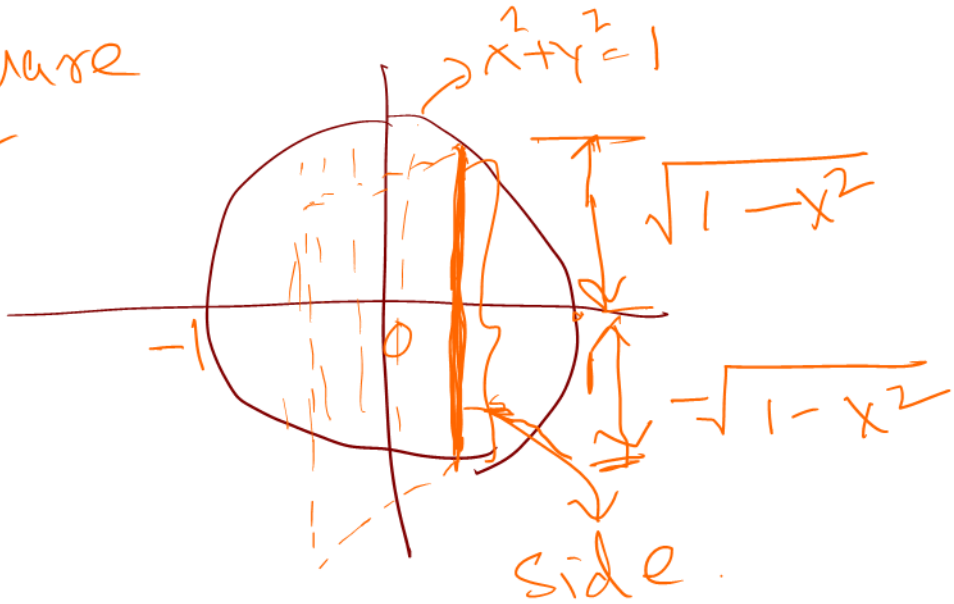
# Lab day 2

The base of a solid is the region bounded by the circle $x^2 + y^2 = 1$. Find the volume of the solid given that the cross sections perpendicular to the x-axis are squares.

Area of a square $= (\text{side})^2$




$\text{side} = 2\left(\sqrt{1-x^2}\right)$

$$\text{Volume} = 2\int_0^1 \left(2\sqrt{1-x^2}\right)^2 dx$$

$$\left(\int_{-1}^{1} \left(2\sqrt{1-x^2}\right)^2 dx\right)$$

$$= 2\int_0^1 4(1-x^2)\,dx$$

$$= 8\left(x - \frac{x^3}{3}\right)\Big|_0^1 = 8\left(1-\frac{1}{3}\right)$$

$$= 16/3$$

Volume: $f(x) = 4x$ $g(x) = x^3$

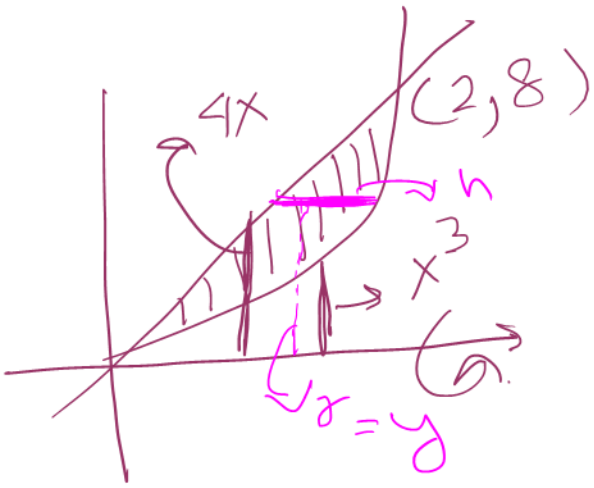
first quardrant

$4x - x^3 = 0$

$x(4 - x^2) = 0$

$x = 0$ $\quad 4 - x^2 = 0$

$x = \pm 2$



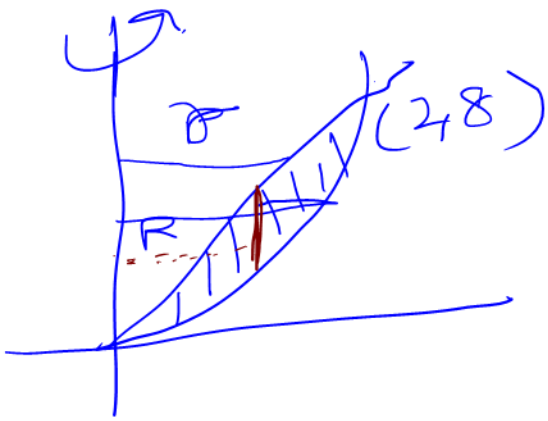

X-axis

Washer: $\pi \int_0^2 (R^2 - r^2)\,dx = \pi \int_0^2 (4x)^2 - (x^3)^2\,dx$

Shell $2\pi \int_0^8 rh\,dy = \int_0^8 2\pi y \downarrow \left(y^{1/3} - \frac{y}{4}\right) dy$

Y-axis

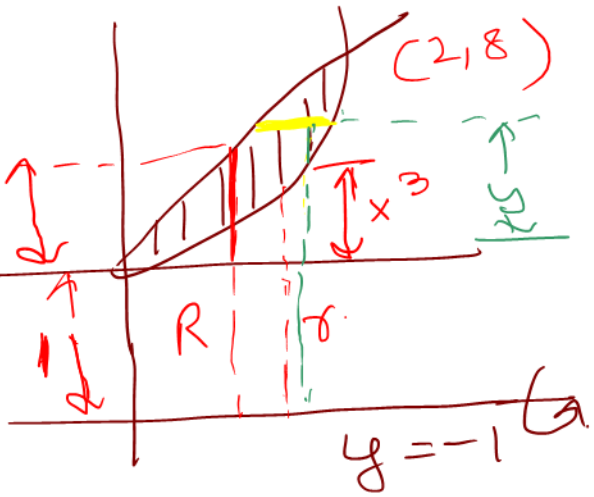
Washer: $\pi \int_0^8 (y^{1/3})^2 - (\tfrac{1}{4}y)^2\,dy$




Shell: $2\pi \int_0^2 x \downarrow (4x - x^3)\,dx$

$f(x) = 4x \quad g(x) = x^3$ 1st quard.

Rotated $y = -1$




Washer:

$$\pi \int_0^2 (1+4x)^2 - (1+x^3)^2 \, dx$$

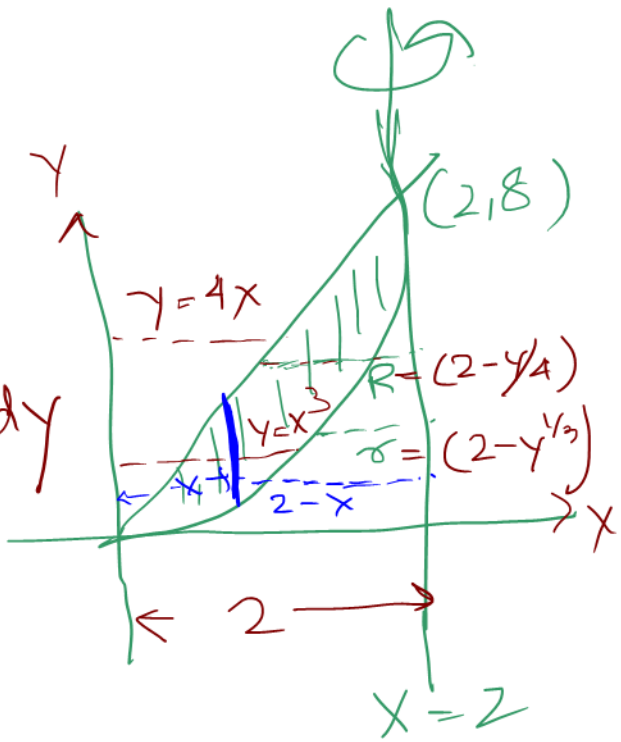
Shell:

$$2\pi \int_0^8 (1+y)\left(y^{1/3} - \frac{y}{4}\right) dy$$

$R = (1+4x)$

$r = (1+x^3)$

Rotated about $x = 2$




Washer:

$$\pi \int_0^8 \left(2 - \frac{y}{4}\right)^2 - \left(2 - y^{1/3}\right)^2 dy$$

Shell:

$$2\pi \int_0^2 (2-x)(4x - x^3)\, dx$$

$f(x) = |4x| \quad [-5, 5]$

$$\text{Avg value} = \frac{1}{5-(-5)} \int_{-5}^{5} |4x| \, dx$$

$$= \frac{1}{10} \int_{-5}^{5} |4x| \, dx$$

$|2| = 2 \quad |-2| = 2 \quad (-(-2) = 2)$

$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

$$\int_{-5}^{5} |4x| \, dx = \int_{-5}^{0} -(4x) \, dx + \int_{0}^{5} 4x \, dx$$

$$= -2x^2 \Big|_{-5}^{0} + 2x^2 \Big|_{0}^{5}$$

$$= -(2 \cdot 0^2 - 2 \cdot (-5)^2) + 2 \cdot 5^2$$

$$= 50 + 50 = 100$$

$$\text{Avg value} = \frac{100}{10} = 10$$