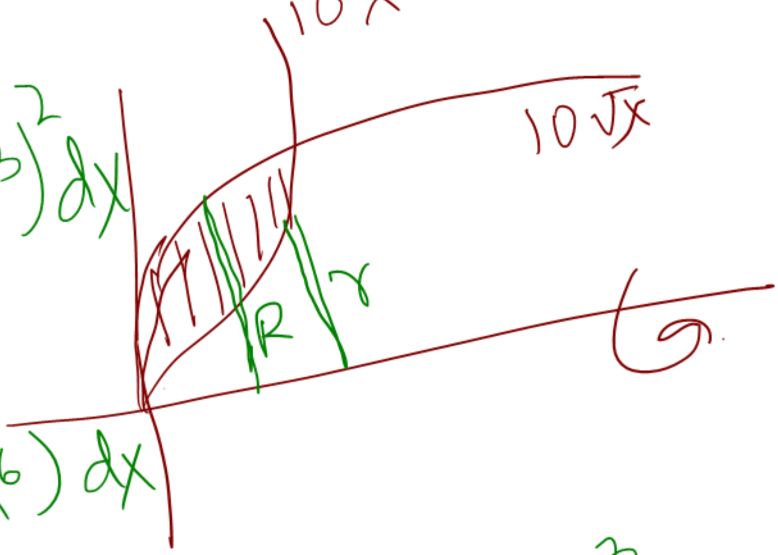
$y = 10\sqrt{x}$ $\quad y = 10x^3$ $\quad$ x-axis



$$V = \pi \int_0^1 (10\sqrt{x})^2 - (10x^3)^2 \, dx$$

$$= \pi \int_0^1 (100x - 100x^6) \, dx$$

$$= 100\pi \left(\frac{x^2}{2} - \frac{x^7}{7} \right) \Big|_0^1$$

$$= 100\pi \left(\frac{1}{2} - \frac{1}{7} \right)$$

$$= 100\pi \left(\frac{7-2}{14} \right)$$

$$= \frac{\cancel{100}^{50}\pi \, 5}{\cancel{14}\,7} = \frac{250\pi}{7}$$

$10\sqrt{x} = 10x^3$

$\sqrt{x} = x^3$

$x = x^6$

$x^6 - x = 0$

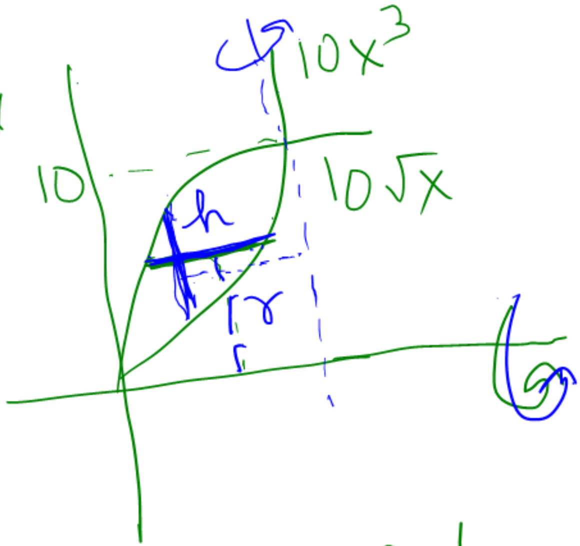
$x(x^5 - 1) = 0$

$x = 0 \quad x^5 - 1 = 0$

$x = 1$

$$V = 2\pi \int_0^{10} y\left(\left(\frac{y}{10}\right)^{1/3} - \frac{y^2}{100}\right) dy$$

$$= 2\pi \int_0^{10} \frac{y^{4/3}}{10^{1/3}} - \frac{y^3}{100}\, dy$$



$$= 2\pi\, \frac{3\, y^{7/3}}{7\, 10^{1/3}} - \frac{y^4}{100\cdot 4}\Big|_0^{10}$$

$x = 0, 1$

$y = 0, 10$

$$= 2\pi\left(\frac{3}{7}\frac{(10)^{7/3}}{10^{1/3}} - \frac{10^4}{10^2\cdot 4}\right)$$

$y = 10\sqrt{x}$

$\frac{y}{10} = \sqrt{x}$

$\frac{y^2}{100} = x$

$$= 2\pi\left(\frac{3}{7}10^{7/3-\frac{1}{3}} - \frac{100}{4}\right)$$

$$= 2\pi\left(\frac{3}{7}10^2 - \frac{100}{4}\right)$$

$y = 10x^3$

$\left(\frac{y}{10}\right)^{1/3} = x$

$$= 200\pi\left(\frac{3}{7} - \frac{1}{4}\right)$$

$$= 200\pi\left(\frac{12-7}{28}\right) = \cancel{200}^{\,\cancel{100}\ 50}\pi\, \frac{5}{\cancel{28}_{\cancel{14}\ 7}}$$

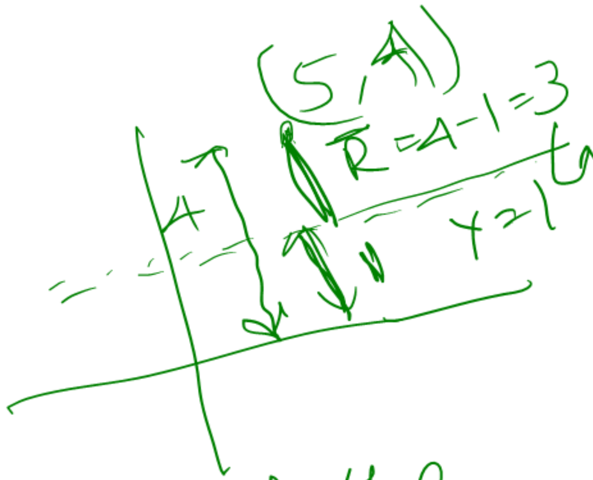
$$= \frac{250\pi}{7}$$

Let a region be in the 1st quad. with centroid (5, 4). Area of the region is 9. Does not interesect the line y=1. Volume rotated y = 1

$$V = 2\pi \bar{R} A$$

$$= 2\pi \cdot 3 \cdot 9$$

$$= 54\pi$$

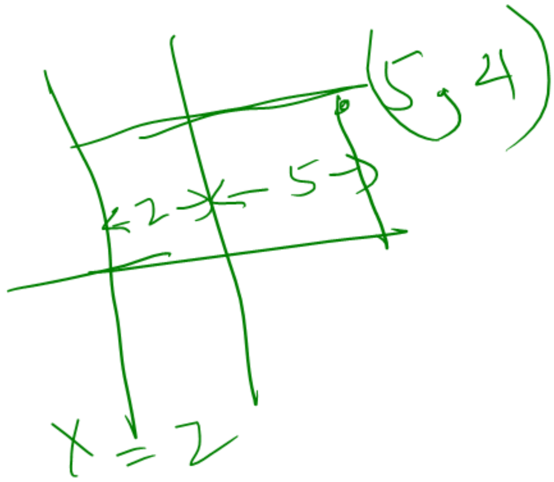


If the region does intersect the line x = -2 what is the volume when rotated x = -2

$$V = 2\pi \bar{R} \cdot A$$

$$= 2\pi \cdot 7 \cdot 9$$

$$= 126\pi$$



$$f(x) = \ln(\sec x) + 5 \quad x \in [0, \tfrac{\pi}{4}]$$

$$\text{Arc length} = \int_a^b \sqrt{1 + (f'(x))^2}\, dx$$

$$f'(x) = \frac{1}{\sec x} \sec x \tan x = \tan x$$

$$\text{Arclength} = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx$$

$$= \ln|\sec x + \tan x| \Big|_0^{\pi/4}$$

$$= \ln\left|\sec\tfrac{\pi}{4} + \tan\tfrac{\pi}{4}\right| - \ln|\sec 0 + \tan 0|$$

$$= \ln|\sqrt{2} + 1| - \ln|1 + 0|$$

$$= \ln|\sqrt{2} + 1|$$

$\underline{\underline{8}}$

$$\frac{dy}{dx} = \frac{e^{x-y}}{1+e^x}$$

$$\frac{dy}{dx} = \frac{e^x \cdot e^{-y}}{1+e^x}$$

$$\int e^y \, dy = \int \frac{e^x}{1+e^x} dx$$

$u = 1+e^x$

$du = e^x dx$

$$e^y = \int \frac{du}{u}$$

$$= \ln|u| + C$$

$$e^y = \ln|e^x| + C$$

$y(1) = 0$

$$e^y = x + C$$

$$e^0 = 1 + C$$

$$1 = 1 + C \Rightarrow C = 0$$

$$e^y = x \Rightarrow y = \ln(x)$$

$$\frac{dy}{dx} = \frac{xy + 2y}{y^4 + 1}$$

$$\frac{dy}{dx} = \frac{y(x+2)}{y^4+1}$$

$$\int \left(\frac{y^4+1}{y}\right) dy = \int (x+2)\, dx$$

$$\int \left(y^3 + \frac{1}{y}\right) dy = \int (x+2)\, dx$$

$$\frac{y^4}{4} + \ln|y| = \frac{x^2}{2} + 2x + C$$

$$y^4 + 4\ln|y| = 2x^2 + 8x + C$$