⊛ $\displaystyle\int_0^{1/2} \frac{3x^2 \, dx}{(1-x^2)^{3/2}}$

$x = \sin\theta$

$dx = \cos\theta \, d\theta$

$(4+x^2)$

$x = 2\tan\theta$

$4 + 4\tan^2\theta$

$4(1+\tan^2\theta)$

$4\sec^2\theta$

$$\int \frac{3x^2 \, dx}{(1-x^2)^{3/2}} = \int \frac{3\sin^2\theta \cos\theta \, d\theta}{(1-\sin^2\theta)^{3/2}}$$

$$= \int \frac{3\sin^2\theta \cancel{\cos}\theta \, d\theta}{\cos^{\cancel{3}\,2}\theta} = 3\int \frac{\sin^2\theta}{\cos^2\theta} \, d\theta$$

$$= 3\int \tan^2\theta \, d\theta$$

$\frac{x}{1} = \sin\theta$

$$= 3\int (\sec^2\theta - 1) \, d\theta$$

$$= 3\left[\int \sec^2\theta \, d\theta - \int d\theta\right]$$

$$= 3\tan\theta - 3\theta + C$$

$$= \frac{3x}{\sqrt{1-x^2}} - 3\sin^{-1}x + C$$

$$\int_0^{1/2} \frac{3x^2 \, dx}{(1-x^2)^{3/2}} = \frac{3x}{\sqrt{1-x^2}} - 3\sin^{-1}x \Big|_0^{1/2}$$

$$= \left(\frac{3 \cdot 1/2}{\sqrt{1-\frac{1}{4}}} - 3\sin^{-1}\frac{1}{2}\right) - \left(\frac{3\,0}{\sqrt{1-0}}^{\to 0} - 3\sin^{-1}0^{\to 0}\right)$$

$$= \frac{3/2}{\sqrt{3/4}} - \frac{3\pi}{6} = \frac{3/\cancel{2}}{\sqrt{3}/\cancel{2}} - \frac{3\pi}{6} = \sqrt{3} - \frac{\pi}{2}$$

$$\int \frac{4}{e^x\sqrt{8+e^{2x}}}\,dx$$

$$e^x = \sqrt{8}\tan\theta$$

$$e^x\,dx = \sqrt{8}\sec^2\theta\,d\theta$$

$$dx = \frac{\cancel{\sqrt{8}}\sec^2\theta\,d\theta}{\sqrt{8}\tan\theta}$$

$$= \int 4\,\frac{\sec^2\theta}{\tan\theta}\,\frac{1}{\sqrt{8+8\tan^2\theta}}\,\frac{d\theta}{\sqrt{8}\tan\theta}$$

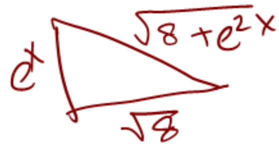
$$= \int 4\,\frac{\sec^{\cancel{2}}\theta}{\tan\theta}\,\frac{1}{\sqrt{8}\,\cancel{\sec\theta}}\,\frac{d\theta}{\sqrt{8}\tan\theta}$$

$$= \int \frac{4}{8}\,\frac{\sec\theta}{\tan^2\theta}\,d\theta = \frac{1}{2}\int \frac{\sec\theta\cos^2\theta}{\sin^2\theta}\,d\theta = \frac{1}{2}\int\frac{\cos\theta}{\sin^2\theta}\,d\theta$$

$$u = \sin\theta$$

$$du = \cos\theta\,d\theta$$

$$e^x = \sqrt{8}\tan\theta$$



$$= \frac{1}{2}\int\frac{du}{u^2} = \frac{1}{2}\int u^{-2}\,du$$

$$= -\frac{1}{2}u^{-1} + C$$

$$= -\frac{1}{2}(\sin\theta)^{-1} + C$$

$$= -\frac{1}{2}\left(\frac{e^x}{\sqrt{8+e^{2x}}}\right)^{-1} + C$$

$$= -\frac{\sqrt{8+e^{2x}}}{e^x} + C$$

$f(x)=\frac{\sqrt{x}(x-3)}{2}$

$$L = \int_9^{64} \sqrt{1 + \left(\tfrac{1}{2}\left(x^{1/2} - x^{-1/2}\right)\right)^2}\, dx$$

$$= \int_9^{64} \sqrt{1 + \tfrac{1}{4}(2x-2)}\, dx$$

$$= \int_9^{64} \sqrt{1 + \tfrac{x}{2} - \tfrac{1}{2}}\, dx$$

$$= \int_9^{64} \sqrt{\tfrac{x}{2} + \tfrac{1}{2}}\, dx$$

$$= \int_9^{64} \sqrt{\tfrac{1}{2}(x+1)}\, dx$$

$$= \int_9^{64} \sqrt{\tfrac{1}{4}\left(x^{1/2}+x^{-1/2}\right)^2}\, dx$$

$$= \frac{1}{2}\int_9^{64} \left(x^{1/2} + x^{-1/2}\right) dx$$

$x^{\frac12}(x-3)$

$$\left(\frac{\sqrt{x}\,(x-3)}{3}\right)' \qquad \frac{d}{dx}x^{3/2} \quad \frac{3}{2}x^{\frac{3}{2}-1}$$

$$= \frac{d}{dx}\left(\frac{x^{3/2} - 3x^{1/2}}{3}\right)$$

$$= \frac{1}{3}\left[\frac{3}{2}x^{1/2} - \frac{3}{2}x^{-1/2}\right]$$

$$= \frac{1}{2}\left[x^{1/2} - x^{-1/2}\right]$$

$$\left(x^{1/2} - x^{-1/2}\right)^2$$

$$= x + x - 2 \quad (x^{1/2})(x^{-1/2})$$

$$= 2x - 2 = 2(x-1)$$

$$\left(x^{1/2} + x^{-1/2}\right)^2$$

$$= x + x + 2$$

$$= 2x + 2 = 2(x+1)$$

$$\frac{\left(x^{1/2} + x^{-1/2}\right)^2}{2} = \cancel{2}(x+1)$$

$$f(x) = \frac{1}{2}x^2 - \ln x$$

$$f'(x) = x - \frac{1}{x} \qquad (f'(x))^2 = \left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$$

$$1 + f'(x)^2 = 1 + x^2 + \frac{1}{x^2} - 2$$

$$= x^2 + \frac{1}{x^2} - 1$$

$$= \left(x - \frac{1}{x}\right)^2$$

$$\sqrt{1 + (f'(x))^2} = x - \frac{1}{x} \qquad L = \int_1^{11} \left(x - \frac{1}{x}\right) dx$$

(*) $$f(x) = \cosh x \qquad f'(x) = \sinh x \qquad f'(x)^2 = \sinh^2 x$$

$$1 + (f'(x))^2 = 1 + \sinh^2 x \qquad \underline{\cosh^2 x - \sinh^2 x = 1}$$

$$= \cosh^2 x$$

$$\sqrt{1 + f'(x)^2} = \cosh x = \frac{e^x + e^{-x}}{2}$$

$$\int_0^{\ln 5} \frac{e^x + e^{-x}}{2} dx = \frac{1}{2} \left| e^x - e^{-x} \right|_0^{\ln 5}$$

$$= \frac{1}{2}\left[\left(e^{\ln 5} - \frac{1}{e^{\ln 5}}\right) - \left(e^0 - e^{-0}\right)\right]$$

$$= \frac{1}{2}\left[\left(5 - \frac{1}{5}\right) - (1 - 1)\right]$$

$$= \frac{\cancel{24}^{12}}{\cancel{2} \cdot 5} = \frac{12}{5}$$

(*) $$f(x) = \frac{(x^2-2)^{3/2}}{3} \qquad f'(x) = \frac{\cancel{3}}{\cancel{2}} \frac{(x^2-2)^{1/2}}{\cancel{3}} (\cancel{2}x) = x(x^2-2)^{1/2}$$

$$(f'(x))^2 = \left(x(x^2-2)^{1/2}\right)^2 = x^2(x^2-2) = x^4 - 2x^2$$

$$1 + f'(x)^2 = 1 + x^4 - 2x^2 = (x^2-1)^2$$

$$\sqrt{1 + f'(x)^2} = x^2 - 1$$

$$\int_2^5 x^2 - 1 \, dx = \frac{x^3}{3} - x \Big|_2^5 = \left(\frac{125}{3} - 5\right) - \left(\frac{8}{3} - 2\right)$$

$$= \frac{125 - 8}{3} - 5 + 2$$

$$= \frac{117}{3} - 3 = 39 - 3 = 36$$

$(*)$ $f(x) = \ln(\sec x)$

P.V $\uparrow$ O.V. $\downarrow$

$P = P_0 e^{kt}$

Half Life

$K \cdot T = \ln\left(\frac{1}{2}\right)$

$T = \dfrac{\ln(1/2)}{\frac{\ln(0.58)}{3}}$

$T = \dfrac{3\ln(1/2)}{\ln(0.58)}$

Example: After 3 days a sample of radon-222 decayed to 58% of its original amount. (a) What is the half-life of radon-222? (b) How long would it take the original sample to decay to 10% of its original amount?

$P_0 = 100 \qquad P = 58$

$58 = 100\, e^{k \cdot 3}$

$0.58 = e^{k \cdot 3} \Rightarrow \ln(0.58) = 3k$

$\Rightarrow k = \dfrac{\ln(0.58)}{3}$

(a) $P = P_0 e^{kt}$ $(P_0 = 100 \quad P = 50)$

$50 = 100\, e^{\frac{\ln(0.58)}{3} t}$

$\frac{1}{2} = e^{\frac{t}{3}\ln(0.58)} \Rightarrow \ln\left(\frac{1}{2}\right) = \frac{t}{3}\ln(0.58)$

$t = \dfrac{3\ln\left(\frac{1}{2}\right)}{\ln(0.58)}$

(b) $P_0 = 100 \qquad P = 10$

$10 = 100\, e^{\frac{\ln(0.58)}{3} t}$

$0.1 = e^{\frac{t}{3}\ln(0.58)} \Rightarrow \ln(0.1) = \frac{t}{3}\ln(0.58)$

$\Rightarrow t = \dfrac{3\ln(0.1)}{\ln(0.58)}$

$$\int_0^1 x^{-2/3}\,dx = \lim_{a\to 0^+}\int_a^1 x^{-2/3}\,dx$$

$$= \lim_{a\to 0^+} \frac{x^{-2/3+1}}{-\frac{2}{3}+1}\Bigg|_a^1$$

$$= \lim_{a\to 0^+} \frac{x^{1/3}}{1/3}\Bigg|_a^1$$

$$= \lim_{a\to 0^+} 3\left[1^{1/3} - a^{1/3}\right] = 3$$

$$\int_0^1 (1-x)^{-2/3}\,dx$$

$$= \lim_{b\to 1^-}\int_0^b (1-x)^{-2/3}\,dx$$

Example: A 100-liter tank initially full of water develops a leak at the bottom. Given that 10% of the water leaks out in the first 5 minutes, find the amount of water left in the tank 15 minutes after the leak develops if the water drains off at a rate that is proportional to the amount of water present.

$$P = P_0 e^{kt} \qquad P_0 = 100, \quad P = 90$$

$$90 = 100\, e^{k5}$$

$$0.9 = e^{5k} \Rightarrow \ln(0.9) = 5k \Rightarrow k = \frac{\ln(0.9)}{5}$$

what is the amount present at any given time t

$$P = 100\, e^{\frac{\ln(0.9)}{5} t} = 100\, e^{\frac{t}{5}\ln(0.9)} = 100\, e^{\ln(0.9)^{t/5}}$$

$$= 100\,(0.9)^{t/5}$$

Now to the ques $t = 15$ $\quad P = 100\,(0.9)^{15/5} = 100\,(0.9)^3$