(*) $\int \frac{3x-15}{x^3+x^2}\,dx$

Q-13,8

$$\frac{3x-15}{x^3+x^2} = \frac{3x-15}{x^2(x+1)} = \frac{Ax+B}{x^2} + \frac{C}{x+1}$$
$$= \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1}$$

$$3x-15 = Ax(x+1) + B(x+1) + Cx^2$$

$x=0$ $\quad -15 = B$ $\qquad B=-15$

$x=-1$ $\quad -18 = A(-1)(-1+1) + B(-1+1) + C(-1)^2 = C$ $\qquad C=-18$

$x=1$ $\quad -12 = A(2) + 2B + C$

$-12 = 2A - 30 - 18 \Rightarrow -12 = 2A - 48$

$\Rightarrow 36 = 2A \Rightarrow A = 18$

$$\int\left(\frac{18}{x} - \frac{15}{x^2} - \frac{18}{x+1}\right)dx = 18\ln|x| + \frac{15}{x} - 18|x+1| + C$$
$$= 18\ln\left|\frac{x}{x+1}\right| + \frac{15}{x} + C$$
$$= -18\ln\left|\frac{x+1}{x}\right| + \frac{15}{x} + C$$

(*) $\int \frac{e^x}{e^{2x}+10e^x+16}\,dx$

Q-13,7D

$u = e^x \Rightarrow du = e^x dx$

$$= \int \frac{du}{u^2+10u+16}$$

$u^2+10u+16$
$= u^2+8u+2u+16$
$= (u+8)(u+2)$

$$= \int \frac{du}{(u+8)(u+2)}$$

$$\frac{1}{(u+8)(u+2)} = \frac{A}{u+8} + \frac{B}{u+2}$$

$$= \int\left(-\frac{1}{6(u+8)} + \frac{1}{6(u+2)}\right)du$$

$1 = A(u+2) + B(u+8)$

$$= -\frac{1}{6}\ln|u+8| + \frac{1}{6}\ln|u+2| + C$$

$u=-2$ $\quad 1 = 6B \Rightarrow B = 1/6$

$u=-8$ $\quad 1 = -6A \Rightarrow A = -1/6$

$$= \frac{1}{6}\ln\left|\frac{u+2}{u+8}\right| + C = \frac{1}{6}\ln\left|\frac{e^x+2}{e^x+8}\right| + C$$

(*) Q-13, 9.9

$$\int \frac{\cos x \, dx}{(\sin x)^2 + 2\sin x - 3}$$

$u = \sin x$

$du = \cos x \, dx$

$$= \int \frac{du}{u^2 + 2u - 3}$$

$u^2 + 2u - 3$

$= u^2 + 3u - u - 3$

$= (u+3)(u-1)$

$$= \int \frac{du}{(u+3)(u-1)}$$

$$\frac{1}{(u+3)(u-1)} = \frac{A}{u+3} + \frac{B}{u-1}$$

$$1 = A(u-1) + B(u+3)$$

$u = 1 \quad 1 = 4B \Rightarrow B = 1/4$

$u = -3 \quad 1 = -4A \Rightarrow A = -1/4$

$$= \int \left(-\frac{1}{4(u+3)} + \frac{1}{4(u-1)} \right) du$$

$$= -\frac{1}{4}\ln|u+3| + \frac{1}{4}\ln|u-1| + C$$

$$= \frac{1}{4}\ln\left|\frac{u-1}{u+3}\right| + C$$

$$= \frac{1}{4}\ln\left|\frac{\sin x - 1}{\sin x + 3}\right| + C$$

(*) Q-10 or -7

$$\int x^3 \cos(4x^2)\,dx$$

$$= \int x^2 \, x \cos(4x^2)\,dx$$

(1) $u = x^2$

$du = 2x\,dx$

(2) $dv = x\cos(4x^2)\,dx$

$V = \int x\cos(4x^2)\,dx$

$= \frac{1}{8}\int \cos s \, ds$

$s = 4x^2$

$= \frac{1}{8}\sin s$

$ds = 8x\,dx$

$= \frac{1}{8}\sin 4x^2$

$\frac{ds}{8} = x\,dx$

$$= \frac{x^2 \sin 4x^2}{8} - \int 2x \frac{1}{8}\sin 4x^2 \, dx$$

$$= \frac{x^2 \sin 4x^2}{8} - \int \frac{x \sin(4x^2)}{4}\,dx$$

$$= \frac{x^2 \sin 4x^2}{8} - \int \frac{\sin u}{8 \cdot 4}\,du$$

(3) $u = 4x^2$

$du = 8x\,dx$

$\frac{du}{8} = x\,dx$

$$= \frac{x^2 \sin 4x^2}{8} + \frac{\cos u}{32} + C$$

$$= \frac{x^2 \sin 4x^2}{8} + \frac{\cos 4x^2}{32} + C$$

Similar Quiz 14, q-13 $\quad n?\quad \varepsilon = 0.01$

$\int_0^4 \cos 4x\,dx \qquad f(x) = \cos 4x$

Trap

$$|E_n^T| \le \frac{(b-a)^3}{12n^2} M$$

$$= \frac{(4-0)^3}{12n^2} 16$$

$$= \frac{64 \cdot \cancel{16}^{4}}{3\,\cancel{12}\,n^2}$$

$$= \frac{85.3333}{n^2}$$

$|f''(x)| \le M \quad x \in [a,b]$

$f''(x) = -16\cos 4x$

$|f''(x)| = |-16\cos 4x|$

≤ 16

$M = 16$

$$\frac{85.3333}{n^2} \le 0.01$$

$$\frac{85.3333}{n^2} \le \frac{1}{100}$$

$$(85.3333)(100) \le n^2$$

$$8533.33 \le n^2$$

$100^2 = 10000$

$90^2 = 8100$

$91^2, \; 92^2 = 8464$

$93^2 = 8649$

$n = 93$

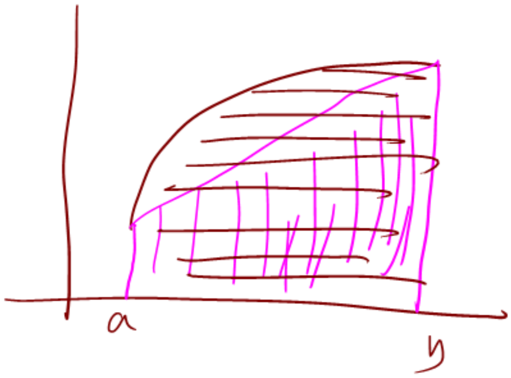
[redacted] Let $f(x)$ be a positive function that is increasing and concave down. Suppose $L_{10}, R_{10}, T_{10}, S_{10}$ are computed to approximate $\int_0^1 f(x)dx$. Use ">, <, or =" symbols to compare the following:

$L_{10} < T_{10} < R_{10}$

$L_{10} < \int_a^b f(x)dx < R_{10}$

$L_{10} < S_{10} < R_{10}$

$T_{10} < \int_a^b f(x)dx$



(*) $\int e^x \cos 2x\,dx$ $\quad u = e^x \quad dv = \cos 2x$

$du = e^x dx \quad v = \frac{\sin 2x}{2}$

$= \frac{e^x \sin 2x}{2} - \int \frac{e^x \sin 2x}{2} dx$

$= \frac{e^x \sin 2x}{2} - \frac{1}{2} \int e^x \sin 2x\,dx$ $\quad u = e^x \quad dv = \sin 2x\,dx$

$du = e^x dx \quad v = \frac{-\cos 2x}{2}$

$= \frac{e^x \sin 2x}{2} - \frac{1}{2}\left[\frac{-e^x \cos 2x}{2} + \int \frac{e^x \cos 2x\,dx}{2}\right]$

$= \frac{e^x \sin 2x}{2} + \frac{1}{4} e^x \cos 2x - \frac{1}{4} \int e^x \cos 2x\,dx$

$\frac{5}{4} \int e^x \cos 2x\,dx = \frac{e^x}{2} \sin 2x + \frac{1}{4} e^x \cos 2x + C$

$\int e^x \cos 2x\,dx = \frac{2e^x \sin 2x}{5} + \frac{2}{5} e^x \cos 2x + C$

$\circledast$ $\int \sin^4 x \cos^5 x\,dx$

$= \int \sin^4 x \cos^4 x \cos x\,dx$

$= \int \sin^4 x (\cos^2 x)^2 \cos x\,dx$

$= \int \sin^4 x (1-\sin^2 x)^2 \cos x\,dx$

$= \int u^4 (1-u^2)^2\,du$

$= \int u^4 (1+u^4-2u^2)\,du$

$= \int (u^4 + u^8 - 2u^6)\,du = \frac{u^5}{5} + \frac{u^9}{9} - \frac{2u^7}{7} + C$

$= \frac{(\sin x)^5}{5} + \frac{(\sin x)^9}{9} - \frac{2(\sin x)^7}{7} + C$

$u = \sin x$

$du = \cos x\,dx$

$\circledast$ $\int \tan^4 x\,dx$

$= \int \tan^2 x \tan^2 x\,dx$

$= \int (\sec^2 x - 1)\tan^2 x\,dx$

$= \int \sec^2 x \tan^2 x\,dx - \int \tan^2 x\,dx$

$= \int \sec^2 x \tan^2 x\,dx - \int (\sec^2 x - 1)\,dx$

$= \int \sec^2 x \tan^2 x\,dx - \int \sec^2 x\,dx + \int dx$

$= \int u^2\,du - \int \sec^2 x\,dx + \int dx$

$= \frac{u^3}{3} - \tan x + x + C = \frac{\tan^3 x}{3} - \tan x + x + C$

$u = \tan x$

$du = \sec^2 x\,dx$

(*) $\int \frac{dx}{(\sqrt{4-x^2})^{3/2}}$

$x = 2\sin\theta$

$dx = 2\cos\theta\, d\theta$

$= \int \frac{2\cos\theta\, d\theta}{(\sqrt{4-4\sin^2\theta})^{3/2}} = \int \frac{2\cos\theta\, d\theta}{(\sqrt{4(1-\sin^2\theta)})^{3/2}}$

$= \int \frac{2\cos\theta\, d\theta}{(2\cos\theta)^{3/2}}$

$= \int \frac{2\cos\theta}{2^{3/2}\cos^{3/2}} d\theta$

$= \int \frac{2^{3/2-1}}{(\cos\theta)^{\frac{3}{2}-1}} d\theta = \int \frac{2^{-1/2}}{(\cos\theta)^{1/2}} d\theta$

$= \int \frac{1}{\sqrt{2\cos\theta}} d\theta$

(*) $\frac{3x-2}{(x+3)(x+2)} = \frac{A}{x+3} + \frac{B}{x+2}$

$\frac{3x-2}{(x+3)^2(x+2)} = \frac{A}{x+3} + \frac{B}{(x+3)^2} + \frac{C}{x+2}$

$\frac{3x-2}{(x^2+3)(x+2)} = \frac{Ax+B}{x^2+3} + \frac{C}{x+2}$