(*) Quiz-17

q-4

$$\sum_{k=5}^{\infty} \frac{3}{k^2-k} = \sum_{5}^{\infty} \frac{3}{k(k-1)} = 3\sum_{5}^{\infty} \frac{1}{k(k-1)}$$

$$= 3\sum_{5}^{\infty} \frac{k-(k-1)}{k(k-1)} \quad \left[\frac{\cancel{k}-\cancel{k}+1}{k(k-1)}\right]$$

$$= 3\sum_{5}^{\infty} \frac{\cancel{k}}{\cancel{k}(k-1)} - \frac{\cancel{k-1}}{k(\cancel{k-1})} \quad \left[\frac{a-b}{c} = \frac{a}{c} - \frac{b}{c}\right]$$

$$= 3\sum_{5}^{\infty} \frac{1}{k-1} - \frac{1}{k}$$

$$\frac{1}{k(k-1)} = \frac{A}{k} + \frac{B}{k-1}$$

$$1 = A(k-1) + Bk$$

$k=0$ $\quad 1 = -A \Rightarrow A = -1$

$k=1$ $\quad 1 = B \Rightarrow B = 1$

$$\frac{1}{k(k-1)} = \frac{1}{k-1} - \frac{1}{k} = 3\left[\left(\frac{1}{4} - \cancel{\frac{1}{5}}\right) + \left(\cancel{\frac{1}{5}} - \cancel{\frac{1}{6}}\right) + \left(\cancel{\frac{1}{6}} - \cancel{\frac{1}{7}}\right) + \cancel{\ } - - \right]$$

$$= \frac{3}{4}$$

q-5

$$\sum_{k=0}^{\infty} \frac{3}{5^k} = 3\sum_{k=0}^{\infty} \left(\frac{1}{5}\right)^k$$

$$= 3\,\frac{1}{1-\frac{1}{5}}$$

$$= 3\,\frac{1}{4/5}$$

$$= \frac{3.5}{4} = \frac{15}{4}$$

A. G.S is conv.

if $|r| < 1$

$1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$

$\left(\frac{1}{2}\right)^0 + \left(\frac{1}{2}\right)^1 + \left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^3, -$

$\sum_{0}^{\infty} \left(\frac{1}{2}\right)^k \quad r = \frac{1}{2}$

$|r| = \frac{1}{2} < 1$

$1, -2, 4, -8, - - -$

$(-2)^0, (-2)^1, (-2)^2, (-2)^3, - - - \quad r = -2$

$|r| = 2$

$$\text{Sum G.S} = \sum_{0}^{\infty} ar^n = \frac{a}{1-r}$$

(*) Quiz-17

q-7

$$\sum_{0}^{\infty} \frac{4^{k+3}}{5^k}$$

$$= \sum_{0}^{\infty} \frac{4^k \cdot 4^3}{5^k} = 64\sum_{0}^{\infty} \left(\frac{4}{5}\right)^k$$

$r = \frac{4}{5} \quad |r| = \frac{4}{5} < 1$

$$= 64\,\frac{1}{1-\frac{4}{5}} = \frac{64}{\frac{1}{5}} = 64.5 = 320$$

$\circledast$ Quiz 17 q-8

$$\sum_{n=1}^{\infty} \frac{5}{n+4}$$

$$s_1 = \frac{5}{1+4} = 1$$

$$s_2 = \frac{5}{1+4} + \frac{5}{2+4} = 1 + \frac{5}{6} = \frac{11}{6}$$

$$s_3 = \frac{5}{1+4} + \frac{5}{2+4} + \frac{5}{3+4} = 1 + \frac{5}{6} + \frac{5}{7} = \frac{42+35+30}{42}$$

$$= \frac{107}{42}$$

Quiz 16 q-7

$$a_n = -6\cos(n\pi)$$

$$6, -6, 6, -6, \ldots$$

q-9

$$a_n = \left(1 + \frac{1}{n}\right)^{5n} \qquad \lim_{n\to\infty}\left(1+\frac{a}{n}\right)^{bn} = e^{ab}$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^{5n} = e^{5}$$

$\circledast$

$$a_n = 2\ln(4n) - \ln(n^2+1)$$

$$= \ln(4n)^2 - \ln(n^2+1)$$

$$= \ln(16n^2) - \ln(n^2+1) = \ln\left(\frac{16n^2}{n^2+1}\right)$$

$$\lim_{n\to\infty} \ln\left(\frac{16n^2}{n^2+1}\right) = \ln\left(\lim_{n\to\infty}\frac{16n^2}{n^2+1}\right) = \ln(16)$$

q-10

$$\lim_{n\to\infty} e^{-8/n} = e^0 = 1$$

q-4

$$\lim_{n\to\infty} \frac{9n}{\sqrt{n^2+1}} = 9$$

q-5

$$\lim_{n\to\infty} \frac{(3n+1)^2}{(5n-1)^2} = \lim_{n\to\infty} \frac{9n^2+1+6n}{25n^2+1-10n} = \frac{9}{25}$$

$$\lim_{n\to\infty} \frac{3n^2+1}{5n^2+2} = \lim_{n\to\infty} \frac{\frac{3n^2}{n^2}+\frac{1}{n^2}}{\frac{5n^2}{n^2}+\frac{2}{n^2}}$$

$$= \lim_{n\to\infty} \frac{3+\frac{1}{n^2}\to 0}{5+\frac{2}{n^2}\to 0} = \frac{3}{5}$$

$$\lim_{n\to\infty} \frac{3n^3+1}{5n^2+2} = \lim_{n\to\infty} \frac{\frac{3n^3}{n^3}+\frac{1}{n^3}}{\frac{5n^2}{n^3}+\frac{2}{n^3}} = \lim_{n\to\infty} \frac{3+\frac{1}{n^3}}{\frac{5}{n}+\frac{2}{n^3}}$$

$$= \frac{3+0}{\underline{\underline{0+0}}} = \text{DNE}$$

$$\lim_{n\to\infty} \frac{3n^2+1}{5n^3+2} = \lim_{n\to\infty} \frac{\frac{3}{n}+\frac{1}{n^3}}{5+\frac{2}{n^3}} = \frac{0+0}{5+0} = \frac{0}{5} = 0$$

$$* \quad \lim_{n\to\infty} n^{1/n} = \lim_{n\to\infty} e^{\ln n^{1/n}} = \lim_{n\to\infty} e^{\frac{1}{n}\ln n}$$

$$= e^{\lim_{n\to\infty} \frac{\ln n}{n}}$$

$$\lim_{n\to\infty} \frac{\ln n}{n} = \lim_{n\to\infty} \frac{1}{n\cdot 1} = 0$$

$$\lim_{n\to\infty} n^{1/n} = e^0 = 1$$

(*) $\left\{-5, -\frac{9}{2}, -\frac{13}{3}, -\frac{17}{4}, \ldots\right\}$

Quiz 15 / Q-3

$$\left\{-\left(\frac{4n+1}{n}\right)\right\}_1^{\infty} = \left\{-\left(4+\frac{1}{n}\right)\right\}_1^{\infty}$$

$\overset{glb}{-5} \rightarrow \overset{lub}{-4}$

(*) Q-7

$a_n = \frac{4}{n}$

$4, \frac{4}{2}, \frac{4}{3}, \frac{4}{4}, \frac{4}{5}$

$4, 2, \frac{4}{3}, 1, \frac{4}{5}, \ldots \rightarrow 0$

$\lim \frac{1}{n} = 0$

$\sum \frac{1}{n}$ div

Quiz 17

q-9

$$\sum \frac{\ln(n)}{n^2+3} \qquad \lim_{n\to\infty} \frac{\ln(n)}{n^2+3} = \lim_{n\to\infty} \frac{1}{n(2n)} = 0$$

n^n
$=$
$n!$
$2^n, e^n$
n^2, n^3

$$\sum \frac{3^n}{n^2} \qquad \lim_{n\to\infty} \frac{3^n}{n^2} = \text{DNE} \not\to 0 \quad \text{Divg by BDT}$$

$$\sum \frac{n!}{(n+3)!} \qquad \lim_{n\to\infty} \frac{n!}{(n+3)!} = \lim_{n\to\infty} \frac{\cancel{1\cdot 2\cdot \cdots n}}{\cancel{1\cdot 2\cdots n}(n+1)(n+2)(n+3)}$$

$$= \lim_{n\to\infty} \frac{1}{(n+1)(n+2)(n+3)} = 0$$

$$\sum \frac{3^n}{n^n} \qquad \lim_{n\to\infty} \frac{3^n}{n^n} = 0$$

BDT

$\lim a_n \not\to 0$

$\sum \frac{1}{n}$

$\lim \frac{1}{n} = 0$

q-10 Conv G. S

$\sum \frac{3}{n!}$ X not G. S

$$\sum \left(\frac{3}{4}\right)^{-n} = \sum \left(\frac{4}{3}\right)^n \qquad r = \frac{4}{3} \quad |r| = \frac{4}{3} > 1$$

$\therefore$ Not Conv

$$\sum \frac{3^{n+1}}{5^n} = \sum \frac{3^n \cdot 3}{5^n} = \sum 3\left(\frac{3}{5}\right)^n \qquad |r| = \frac{3}{5} < 1 \quad \therefore \text{Conv}$$

$$\sum (1.03)^n \qquad |r| = 1.03 > 1 \quad \text{Not Conv}$$

Quiz - 17
Q - 1

$$-1 + 5 + 11 + 17 + \cdots + 59$$

$6 \cdot 0 - 1$ $\quad 6 \cdot 1 - 1$ $\quad 6 \cdot 2 - 1$ $\quad 6 \cdot 3 - 1$ $\quad 6 \cdot 10 - 1$

$$\sum_{k=0}^{10} 6k - 1$$

Quiz - 13
Q - 5

$$\int \frac{1}{4x^2 + 8x + 8} dx$$

$$= \frac{1}{4} \int \frac{1}{x^2 + 2x + 2} dx$$

$$= \frac{1}{4} \int \frac{1}{1 + (x+1)^2} dx$$

$$= \frac{1}{4} \int \frac{1}{1 + u^2} du$$

$$= \frac{1}{4} \tan^{-1} u + C = \frac{1}{4} \tan^{-1} x+1 + C$$

$$x^2 + 2x + 2$$
$$= (x^2 + 2x + 1) + 1$$
$$= (x+1)^2 + 1$$

$$u = x + 1$$
$$du = dx$$

quiz 13

q-9

$$\int \frac{\cos x}{(\sin x)^2+2\sin x-3}\,dx$$

$u = \sin x$

$du = \cos x\,dx$

$$=\int \frac{du}{u^2+2u-3}$$

u^2+2u-3

$=u^2+3u-u-3$

$=(u+3)(u-1)$

$$=\int \frac{du}{(u+3)(u-1)}$$

$$\frac{1}{(u+3)(u-1)} = \frac{A}{u+3} + \frac{B}{u-1}$$

$1 = A(u-1) + B(u+3)$

$u=1 \quad 1 = 4B \Rightarrow B = 1/4$

$u=-3 \quad 1 = 2A \Rightarrow A = 1/2$

$$= \int \left(\frac{1}{2(u+3)} + \frac{1}{4(u-1)}\right) du$$

$$= \frac{1}{2}\ln|u+3| + \frac{1}{4}\ln|u-1| + C$$

$$= \frac{1}{2}\ln|\sin x+3| + \frac{1}{4}\ln|\sin x-1| + C$$

$$= \frac{1}{2}\left[\ln\left|\frac{\sin x-3}{\sin x-1}\right|\right] + C.$$

q-17

$$\int \frac{e^x}{e^{2x}+7e^x+10}\,dx$$

$u = e^x \Rightarrow du = e^x\,dx$

$$=\int \frac{du}{u^2+7u+10}$$

$u^2+7u+10$

$=u^2+5u+2u+10$

$=(u+5)(u+2)$

$$=\int \frac{1}{(u+5)(u+2)}\,du$$