Quiz - 17

Q - 4

$$\sum_{K=1}^{\infty} \frac{1}{K^2+4K} = \sum_{K=1}^{\infty} \frac{1}{K(K+4)}$$

$$\frac{1}{K(K+4)} = \frac{A}{K} + \frac{B}{K+4}$$

$$1 = A(K+4) + BK$$

$K=0 \quad 1 = 4A \Rightarrow A = \frac{1}{4}$

$K=-4 \quad 1 = -4B \Rightarrow B = -\frac{1}{4}$

$$\sum_{1}^{\infty} \frac{1}{K(K+4)} = \sum_{1}^{\infty} \frac{1}{4}\left[\frac{1}{K} - \frac{1}{K+4}\right]$$

$$= \frac{1}{4}\left[\left(1 - \cancel{\tfrac{1}{5}}\right) + \left(\tfrac{1}{2} - \cancel{\tfrac{1}{6}}\right) + \left(\tfrac{1}{3} - \cancel{\tfrac{1}{7}}\right) + \left(\tfrac{1}{4} - \cancel{\tfrac{1}{8}}\right) + \left(\cancel{\tfrac{1}{5}} - \cancel{\tfrac{1}{9}}\right) + \left(\cancel{\tfrac{1}{6}} - \cancel{\tfrac{1}{10}}\right) + \cdots\right]$$

$$= \frac{1}{4}\left[\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right)\right]$$

$$= \frac{1}{4}\left[\frac{12+6+4+3}{12}\right] = \frac{25}{48}$$

Q - 6

$$\sum_{3}^{\infty} \frac{5}{9^K} = 5 \sum_{3}^{\infty} \left(\frac{1}{9}\right)^K$$

$r = \frac{1}{9} \quad |r| = \frac{1}{9} < 1$

$\therefore$ the G.S is conv gt

$$G.S = \frac{a}{1-r}$$

$$\left(\frac{1}{9}\right)^3 + \left(\frac{1}{9}\right)^4 + \left(\frac{1}{9}\right)^5 + \cdots$$

$$\left(\frac{1}{9}\right)^3 + \left(\frac{1}{9}\right)^3\left(\frac{1}{9}\right) + \left(\frac{1}{9}\right)^3\left(\frac{1}{9}\right)^2$$

$$= 5 \frac{(1/9)^3}{1 - \frac{1}{9}}$$

$$= 5 \frac{\frac{1}{81 \cdot \cancel{9}}}{8/\cancel{9}} = \frac{5}{648}$$

Suppose

$$\sum_{n=0}^{\infty} \frac{(-2)^{3n+2}}{5^n} = \sum_{n=0}^{\infty} \frac{(-2)^{3n}(-2)^2}{5^n} = \sum_{n=0}^{\infty} \left(\frac{-8}{5}\right)^n 4$$

$r = -\frac{8}{5} \quad |r| = \left|-\frac{8}{5}\right| = \frac{8}{5} > 1$

$\therefore$ This G.S is Div.

Q17 q-8

$$\sum_{k=1}^{\infty}\left(\frac{1}{6}\right)^k$$

$$S_1 = \left(\frac{1}{6}\right)^1 = \frac{1}{6}$$

$$S_2 = \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 = \frac{1}{6} + \frac{1}{36} = \frac{6+1}{36} = \frac{7}{36}$$

$$S_3 = \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3$$

$$S_4 = \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4$$

$$S_5 = \left(\frac{1}{6}\right)^1 + \left(\frac{1}{6}\right)^2 + \left(\frac{1}{6}\right)^3 + \left(\frac{1}{6}\right)^4 + \left(\frac{1}{6}\right)^5$$

q-9

$$\sum_{1}^{\infty}\frac{\sqrt{n}}{n^2+2} \qquad \lim_{n\to\infty}\frac{\sqrt{n}}{n^2+2} = 0$$ So we cannot say it diverges By BDT

$$\sum_{1}^{\infty}\frac{(-1)^n}{n^2} \qquad \lim_{n\to\infty}\frac{(-1)^n}{n^2} = 0$$ "

$$\sum_{1}^{\infty}\frac{n+2}{(n+3)!} \qquad \lim_{n\to\infty}\frac{n+2}{(n+3)!} = \lim_{n\to\infty}\frac{n+2}{1\cdot 2\cdot 3\cdots n(n+1)(n+2)(n+3)} = 0$$

$$\sum_{1}^{\infty}\frac{n^3}{n^2+2} \qquad \lim_{n\to\infty}\frac{n^3}{n^2+2} = \text{DNE} \not\to 0$$

$\therefore$ The series is Divgt by BDT

Q-18
q-17

(*) $$\sum \frac{4+\sin k}{5k^2} = \sum \frac{4}{5k^2} + \sum \frac{\sin k}{5k^2}$$
$$= \frac{4}{5}\sum \frac{1}{k^2} + \frac{1}{5}\sum \frac{\sin k}{k^2}$$

$\sum \frac{1}{k^2}$ is convgt by p-series test as $p=2>1$

$\sum \frac{\sin k}{k^2} \le \sum \frac{1}{k^2} \to$ is conv by p series test as $p=2>1$ $\left[\ \sin k < 1\right.$

$\therefore$ By BCT $\sum \frac{\sin k}{k^2}$ is convgt

OR

$$\sum \frac{4+\sin k}{5k^2} \le \sum \frac{4+1}{5k^2} = \sum \frac{5}{5k^2} = \sum \frac{1}{k^2}$$

$\sum \frac{1}{k^2}$ is convgt by p-series test as $p=2>1$

$\therefore$ By BCT $\sum \frac{4+\sin k}{5k^2}$ is conv

(*) Q-18
q-3

$$\sum \frac{\arctan 4k}{16k^2+1} \qquad a_n = \frac{\tan^{-1} 4k}{16k^2+1}$$

(i) a_n is positive
(ii) a_n is decreasing
(iii) a_n is continuous

$$\int_1^\infty \frac{\tan^{-1}4x}{16x^2+1}\,dx = \lim_{b\to\infty}\int_1^b \frac{\tan^{-1}4x}{16x^2+1}\,dx$$

$$\int \frac{\tan^{-1}4x}{16x^2+1}\,dx = \frac{1}{4}\int u\,du = \frac{u^2}{8}+C \qquad u=\tan^{-1}4x \qquad du = \frac{4\,dx}{16x^2+1}$$
$$= \frac{(\tan^{-1}4x)^2}{8}+C$$

$$\lim_{b\to\infty}\int_1^b \frac{\tan^{-1}4x}{16x^2+1}\,dx = \lim_{b\to\infty}\frac{(\tan^{-1}4x)^2}{8}\Big|_1^b = \lim_{b\to\infty}\frac{(\tan^{-1}4b)^2}{8} - \frac{(\tan^{-1}4)^2}{8} = \frac{\left(\frac{\pi}{2}\right)^2}{8} - \frac{(\tan^{-1}4)^2}{8}$$

$\therefore$ By Integral test the series is conv

(*) q-10 $\sum_{1}^{\infty} \frac{(-1)^n}{4^n} = \sum \left(-\frac{1}{4}\right)^n$ $r = -\frac{1}{4}$ $|r| = \frac{1}{4} < 1$ conv by G.S test

$\sum_{1}^{\infty} \frac{5^n}{4^n} = \sum \left(\frac{5}{4}\right)^n$ $r = \frac{5}{4}$ $|r| = \frac{5}{4} > 1$ Div

$\sum_{1}^{\infty} (1.04)^n$ $r = 1.04$ $|r| = 1.04 > 1$ Div by G.S test

$\sum_{1}^{\infty} \frac{4}{n^2} \to$ is not G S

Quiz 18
q-10

(*) $\sum k\left(\frac{3}{5}\right)^k$

$\lim_{k\to\infty} \left(k\left(\frac{3}{5}\right)^k\right)^{1/k}$

$= \lim_{k\to\infty} k^{1/k} \frac{3}{5} = \frac{3}{5} < 1$

$\therefore$ Convergent by Root test

q-4

$\sum \frac{\ln\sqrt{4+k}}{6k} \sim \sum \frac{1}{k}$

$\ln n < n$

$\frac{\ln n}{n} < 1$

q-6 $\sum \frac{4ke^{-k^2}}{5} = \frac{4}{5}\sum \frac{k}{e^{k^2}}$

$\lim_{k\to\infty} \left(\frac{k}{e^{k^2}}\right)^{1/k} = \lim_{k\to\infty} \frac{k^{1/k}}{e^{k^2 \cdot \frac{1}{k}}} = \lim_{k\to\infty} \frac{k^{1/k} \to 1}{e^k \to \infty} = 0 < 1$

$\therefore$ Convgt by Root test

⊛ Q-18
9-5

$$\sum \frac{k^4-1}{3k^2+5} < \sum \frac{k^4-1}{3k^2}$$

$\frac{3}{5}=0.6$ $\frac{3}{2}=1.5$

$\frac{3}{2}=1.5$ $\frac{4}{2}=2$

$$\left(\sum \frac{k^4-1}{3k^2+5} \cancel{<} \sum \frac{k^4}{3k^2}\right) < \sum \frac{k^4+k^4}{3k^2}$$

$$= \sum \frac{2k^4}{3k^2}$$

$$= \sum \frac{2}{3}k^2$$

$$= \sum \frac{k^4}{3k^2} - \frac{1}{k^2}$$

DIV

$$= \sum \frac{k^2}{3} - \frac{1}{k^2} \rightarrow \text{Conv } p>1$$

⊛ Emcf 9.5

Q5

$$\sum 3n^{-2/3} = \sum \frac{3}{n^{2/3}}$$

$p = \frac{2}{3} < 1$ ∴ the series is div by p-series test

(*) Emcf 9.4

12-5 $\sum a_n = \sum \frac{n^2}{n^3 - 2n + 5}$ $\qquad \sum b_n = \sum \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{\frac{n^2}{n^3-2n+5}}{\frac{1}{n}}$$

$$= \lim_{n\to\infty} \frac{n^3}{n^3-2n+5} = 1$$

$\sum b_n$ is div as $p=1$ by p-series test

$\therefore$ By LCT $\sum a_n$ is div

(*) 9.2 Emcf 9.5 $\sum \frac{n!}{10^n}$ $\qquad a_n = \frac{n!}{10^n}$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)!}{10^{n+1}} \cdot \frac{10^n}{n!}$$

$$\frac{\cancel{1}\cdot\cancel{2}\cdot - \cancel{n}\, n+1}{\cancel{1}\cdot\cancel{2}\cdot - \cancel{n}}$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \frac{10^n}{10^{n+1}}$$

$$= \lim_{n\to\infty} \frac{n+1}{10} = \text{DNE} > 1$$

$\therefore$ Div by Ratio test