(*) Qu-18 / q-9

$$\sum \frac{k^2+2}{k(k^2+4)} = \sum \frac{k^2+2}{k^3+4k}$$

$$\sum a_k = \sum \frac{k^2+2}{k^3+4k} \quad \text{and} \quad \sum b_k = \sum \frac{1}{k} \quad \left(\sum k^{2-3} = \sum k^{-1} = \sum \frac{1}{k}\right)$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^3+2k}{k^3+4k} = 1 > 0$$

Now $\sum \frac{1}{k}$ is div by p-series test as $p=1$

$\therefore$ By LCT $\sum a_k$ is divergent.

(*) Q-17 / q-2

$$3\cdot 5 + 4\cdot 6 + 5\cdot 7 + \cdots$$

$$(0+3)(0+5) + (1+3)(1+5) + (2+3)(2+5) + \cdots$$

$$\sum_{k=0}^{\infty} (k+3)(k+5) \qquad (1+2)(1+4) + (2+2)(2+4) + \cdots$$

$$\sum_{k=1}^{\infty} (k+2)(k+4)$$

(*) q-3

$$\frac{1}{5^4} + \frac{1}{5^5} + \frac{1}{5^6} + \cdots + \frac{1}{5^{11}}$$

$$\sum_{k=4}^{11} \frac{1}{5^k} \quad \& \quad \sum_{i=0}^{7} \frac{1}{5^{i+4}} \checkmark$$

$$\times \sum_{k=4}^{11} \frac{1}{5^{k+4}} \times \quad \& \quad \sum_{i=0}^{7} \frac{1}{5^{i+4}} \checkmark \qquad \times \sum_{k=1}^{11} \frac{1}{5^k} \quad \& \quad \sum_{i=4}^{7} \frac{1}{5^{i+4}} \times$$

$$\frac{1}{5^4} + \frac{1}{5^8} + \frac{1}{5^{10}} + \frac{1}{5^7}$$

(*) $$\sum_{k=0}^{\infty} \frac{1-3^k}{6^k} = \sum_{0}^{\infty}\left(\left(\frac{1}{6^k}\right)-\left(\frac{3^k}{6^k}\right)\right)$$

$$= \sum_{0}^{\infty} \left(\frac{1}{6}\right)^k - \sum_{0}^{\infty} \left(\frac{3}{6}\right)^k$$

$$= \sum_{0}^{\infty} \left(\frac{1}{6}\right)^k - \sum_{0}^{\infty} \left(\frac{1}{2}\right)^k$$

$r = \frac{1}{6}$ $\quad |r| = \frac{1}{6} < 1$ $\qquad\qquad$ $r = \frac{1}{2}$ $\quad |r| = \frac{1}{2} < 1$

$$\text{Sum} = \frac{\left(\frac{1}{6}\right)^0}{1-\frac{1}{6}} \qquad\qquad \frac{\left(\frac{1}{2}\right)^0}{1-\frac{1}{2}}$$

$$= \frac{1}{5/6} = \frac{6}{5} \qquad\qquad = \frac{1}{1/2} = 2$$

$$= \frac{6}{5} - 2 = \frac{6-10}{5} = -\frac{4}{5}$$

(*) $$\sum_{k=1}^{\infty} \frac{n+6}{n^2}$$

$$S_1 = \frac{1+6}{1^2} = 7$$

$$S_2 = \frac{1+6}{1^2} + \frac{2+6}{2^2} = 7 + \frac{8}{4} = 9$$

$$S_3 = \frac{1+6}{1^2} + \frac{2+6}{2^2} + \frac{3+6}{3^2} = 7+2+1 = 10$$

$$S_4 = \frac{1+6}{1^2} + \frac{2+6}{2^2} + \frac{3+6}{3^2} + \frac{4+6}{4^2} = 7+2+1+\frac{10}{16} = 10+\frac{5}{8} = \frac{85}{8}$$

11-9 Q-17

$\sum \frac{\ln(n)}{n^2+6}$ $\lim_{n\to\infty} \frac{\ln n}{n^2+6} = \lim_{n\to\infty} \frac{1}{n(2n)} = 0$

$\sum \frac{6^n}{n^2}$ $\lim_{n\to\infty} \frac{6^n}{n^2} = DNE \not\to 0$

$\sum \frac{6^n}{n^n}$ $\lim_{n\to\infty} \frac{6^n}{n^n} = 0$

$\sum \frac{n!}{(6+n)!} = \sum \frac{1.2 \ldots n}{1.2 \ldots n(n+1)(n+2)\ldots(n+6)}$

$= \sum \frac{1}{(n+1)(n+2)\ldots(n+6)}$ $\lim \frac{1}{(n+1)(n+2)\ldots(n+6)} = 0$

n^n
$\downarrow$
$n!$
$\downarrow$
$2^n, e^n$
$\downarrow$
n^2, n^3

$\sum \frac{6^{n+1}}{7^n} = \sum \frac{6^n \cdot 6}{7^n} = \sum \left(\frac{6}{7}\right)^n \cdot 6$ $r = \frac{6}{7}$ $|r| = \frac{6}{7} < 1$

$\Rightarrow$ Conv

$\sum \frac{6}{n!} \to$ Not G.S

$\sum \left(\frac{3}{4}\right)^{-n} = \sum \left(\frac{4}{3}\right)^n$ $r = \frac{4}{3}$ $|r| = \frac{4}{3} > 1 \Rightarrow$ Div

$\sum (1.06)^n$ $r = 1.06$ $|r| = 1.06 > 1 \Rightarrow$ Div

Q-15
Q-5

$$\sum \frac{(-1)^k \ln k}{k+3}$$

$$\sum \left|\frac{(-1)^k \ln k}{k+3}\right| = \sum \frac{\ln k}{k+3}$$

$$\sum \frac{\ln k}{k+3} \geq \sum \frac{1}{k+3} \geq \sum \frac{1}{k+k} = \sum \frac{1}{2k} \text{ div as } p=1$$

$\therefore$ BCT $\sum \frac{\ln k}{k+3}$ is div.

Hence $\sum \frac{(-1)^k \ln k}{k+3}$ is not abs conv

$a_k = \frac{\ln k}{k+3}$ (i) continuous

(ii) mono dec

(iii) $\lim_{k\to\infty} a_k = \lim_{k\to\infty} \frac{\ln k}{k+3} = \lim_{k\to\infty} \frac{1}{k \cdot 1} = 0$

$\therefore$ The series is conv by alt. series test.

Hence the series is cond. conv.

Q-19
Q-2

$$\sum \frac{5k!}{(k+5)!} \qquad \sum \frac{(5k)!}{(k+5)!}$$

$$= 5\sum \frac{k!}{k+5!}$$

$$= 5\sum \frac{1.2 \ldots k}{1.2 \ldots k(k+1)(k+2)\ldots(k+5)}$$

$$= 5\sum \frac{1}{(k+1)(k+2)(k+3)(k+4)(k+5)} < 5\sum \frac{1}{k^5}$$

$\sum \frac{1}{k^5}$ is conv by p-series test as $p=5>1$

$\therefore$ BCT the series is conv.

Q-9

$$\sum \left| \frac{(-1)^k (k+7)}{k^2+3k+1} \right| = \sum \frac{k+7}{k^2+3k+1}$$

$$\sum a_k = \sum \frac{k+7}{k^2+3k+1} \qquad \sum b_k = \sum \frac{1}{k}$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k^2+7k}{k^2+3k+1} = 1 > 0$$

$\sum b_k$ is div as $p=1$ $\therefore$ $\sum a_k$ is div by LCT

Hence the series is not abs conv

$a_k = \dfrac{k+7}{k^2+3k+1}$ (i) continuous

(ii) mono dec.

(iii) $\lim_{k\to\infty} \dfrac{k+7}{k^2+3k+1} = 0$

$\therefore$ The series is convgt by alt series test

Hence the series is cond. conv.