Quiz-20

q-9

$$\sum \frac{k^3 (x-8)^k}{e^k}$$

$$\lim_{k\to\infty} \left| \left(\frac{k^3 (x-8)^k}{e^k} \right)^{\frac{1}{k}} \right|$$

$$= \lim_{k\to\infty} \left| \frac{\overset{\to 1}{k^{3/k}} (x-8)}{e} \right| = \frac{|x-8|}{e}$$

For convergence by roots test

$$\frac{|x-8|}{e} < 1 \Rightarrow |x-8| < e$$

$$\Rightarrow -e < x-8 < e$$

$$\Rightarrow 8-e < x < e+8$$

$x = 8-e$

$$\sum \frac{k^3 (x-8)^k}{e^k}$$

$$= \sum \frac{k^3 (\cancel{8}-e-\cancel{8})^k}{e^k} = \sum \frac{k^3 (-e)^k}{e^k}$$

$(-1.e)^k$

$$= \sum \frac{(-1)^k k^3 \cancel{e^k}}{\cancel{e^k}} = \sum (-1)^k k^3$$

Divergent

$x = e+8$

$$\sum \frac{k^3 (e+\cancel{8}-\cancel{8})^k}{e^k} = \sum \frac{k^3 e^k}{e^k}$$

$$= \sum k^3 \text{ Divergent}$$

$(8-e, 8+e)$

Q-10 $\sum \frac{(-1)^k (k+5) x^4}{3^k}$

$$\lim_{k\to\infty} \left| \left(\frac{(-1)^k (\cancel{k}+5) x^4}{3^k} \right)^{1/k} \right| = \lim_{k\to\infty} \left| \frac{(-1) \overbrace{(k+5)^{1/k}}^{\to 1} x}{3} \right|$$

$$= \left| \frac{(-1) x}{3} \right|$$

$$= \frac{|x|}{3}$$

$\frac{|x|}{3} < 1 \Rightarrow |x| < 3 \Rightarrow -3 < x < 3$

at $x = 3$ $\quad \sum \frac{(-1)^k (k+5) 3^4}{3^k} = \sum (-1)^k (k+5)$ DIV (-1.3)

at $x = -3$ $\quad \sum \frac{(-1)^k (k+5) (-3)^4}{3^k}$

$$= \sum \frac{(-1)^k (k+5) (-1)^4 \cancel{3^k}}{\cancel{3^k}}$$

$$= \sum (-1)^{2k} (k+5) = \sum (k+5) \text{ DIV}$$

Intv $(-3, 3)$

(*)

9-11

$$\sum \frac{(-1)^k k!(x-3)^k}{(k+5)^3} \quad F(3)=0$$

$$\int \sum \frac{(-1)^k k!(x-3)^k}{(k+5)^3}\,dx$$

$$F(x) = \sum \frac{(-1)^k k!(x-3)^{k+1}}{(k+5)^3(k+1)} + C$$

$$0 = \sum \frac{(-1) k!(3-3)^{k+1}}{(k+5)^3(k+1)} + C$$

$$\Rightarrow C=0$$

$$F(x) = \sum \frac{(-1)^k k!(x-3)^{k+1}}{(k+5)^3(k+1)}$$

(*)

$$\frac{d}{dx}\sum \frac{x^k}{(k+3)4^k}$$

$$= \sum \frac{k\,x^{k-1}}{(k+3)4^k}$$

Midterm A

q-2

$$\int_0^a 6x\sqrt{a^2-x^2}\,dx$$

$$\int 6x\sqrt{a^2-x^2}\,dx$$

$u = a^2 - x^2$

$du = -2x\,dx$

$-du = 2x\,dx$

$$= -3\int \sqrt{u}\,du$$

$$= -3\,\frac{u^{\frac{1}{2}+1}}{\frac{1}{2}+1} + C$$

$$= -\cancel{3}\,\frac{u^{3/2}}{\cancel{3}/2} + C$$

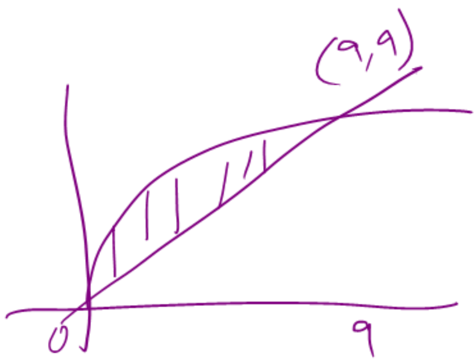
$$= -2u^{3/2} + C = -2(a^2-x^2)^{3/2}$$

$$\int_0^a 6x\sqrt{a^2-x^2}\,dx = -2(a^2-x^2)^{3/2}\Big|_0^a$$

$$= -2(a^2-a^2)^{3/2} + 2(a^2-0^2)^{3/2}$$

$$= 2a^3$$

* q-6 $\quad y = 3\sqrt{x} \qquad y = x$



$$3\sqrt{x} = x$$

$$9x = x^2$$

$$x^2 - 9x = 0 \Rightarrow x = 0, 9$$

$$\int_0^9 (3\sqrt{x} - x)\,dx = \cancel{3}\,\frac{x^{3/2}}{\cancel{3}/2} - \frac{x^2}{2}\Big|_0^9$$

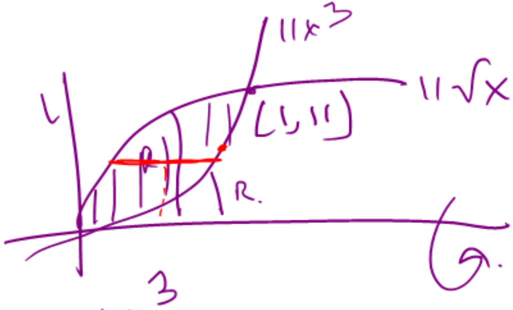
$$= 2x^{3/2} - \frac{x^2}{2}\Big|_0^9$$

$$= 2\cdot 27 - \frac{81}{2}$$

$$= \frac{108 - 81}{2} = \frac{27}{2}$$

(*) Q-8 $y = 11\sqrt{x}$ $y = 11x^3$

about x-axis



$$11\sqrt{x} = 11x^3 \Rightarrow \sqrt{x} = x^3$$
$$\Rightarrow x = x^6$$
$$\Rightarrow x^6 - x = 0$$
$$\Rightarrow x(x^5 - 1) = 0$$
$$\Rightarrow x = 0, 1$$

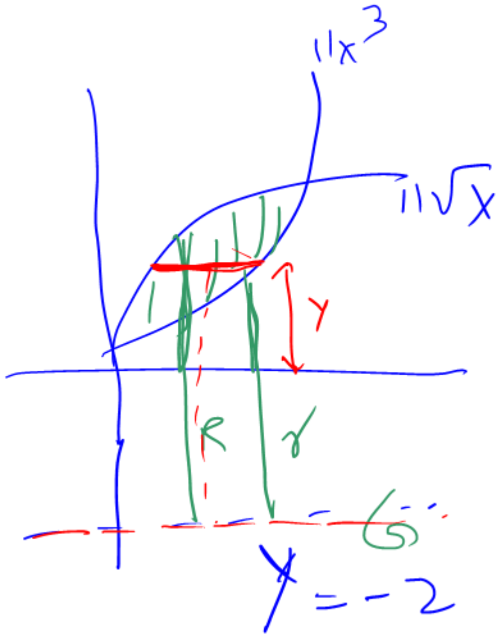
$y = 11x^3$
$\left(\frac{y}{11}\right)^{1/3} = x$

$y = 11\sqrt{x}$
$\left(\frac{y}{11}\right)^2 = x$

$$\pi \int_0^1 \left((11\sqrt{x})^2 - (11x^3)^2 \right) dx$$

$$2\pi \int_0^{11} y \left(\left(\frac{y}{11}\right)^{1/3} - \left(\frac{y}{11}\right)^2 \right) dy$$

About $y = -2$

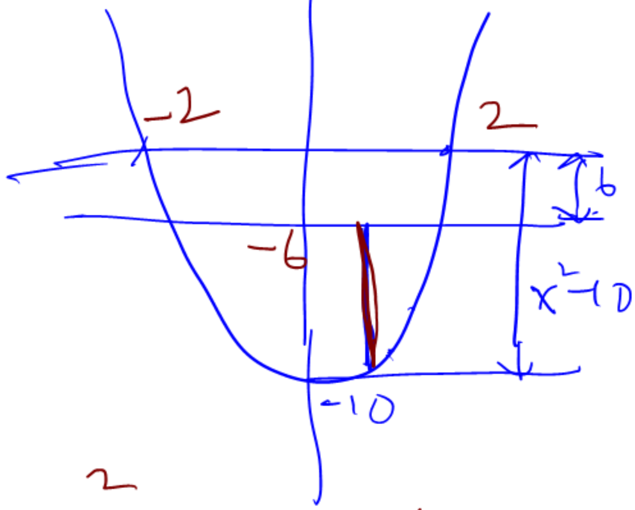


$$\pi \int_0^1 \left((11\sqrt{x} + 2)^2 - (11x^3 + 2)^2 \right) dx$$

$$2\pi \int_0^{11} (y+2) \left(\left(\frac{y}{11}\right)^{1/3} - \left(\frac{y}{11}\right)^2 \right) dy$$

CORRECTED

(*) 9-9 $y = x^2 - 10$ $\quad y = 6$



$\text{Side} = -6 - (x^2 - 10)$

$= -6 - x^2 + 10$

$= 4 - x^2$

$\text{Area} = (4 - x^2)^2$

$\text{volume} = \int_{-2}^{2} (4 - x^2)^2 \, dx$

OR $= 2 \int_{0}^{2} (4 - x^2)^2 \, dx$

$x^2 - 10 = -6$

$x^2 = 10 - 6$

$x^2 = 4$

$x = \pm 2$

(*) 9-18

$P = P_0 e^{kt}$

$6 = 7 e^{2k}$ (2 years ago)

$\frac{6}{7} = e^{2k} \Rightarrow \ln\left(\frac{6}{7}\right) = 2k$

$\Rightarrow k \equiv \frac{1}{2} \ln\left(\frac{6}{7}\right)$

2 years forward.

$P = 7 e^{\frac{1}{2}\ln\left(\frac{6}{7}\right) \cdot 4 \cdot 2}$

$= 7 e^{2 \ln\left(\frac{6}{7}\right)}$

$= 7 e^{\ln\left(\frac{6}{7}\right)^2}$

$= 7 \cdot \left(\frac{6}{7}\right)^2 = \frac{36}{7} = 5.142$

OR

$P = 6 e^{\frac{1}{2}\ln\left(\frac{6}{7}\right) 2}$

$= 6 e^{\ln\left(\frac{6}{7}\right)}$

$= 6 \cdot \left(\frac{6}{7}\right)$

Quiz - 21

q - 3

$f(x) = \sinh 6x$

$\sinh x = x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots$

$\sinh x = \sum \frac{x^{2k+1}}{(2k+1)!}$

$\sinh 6x = \sum \frac{(6x)^{2k+1}}{(2k+1)!}$

$= \sum \frac{6^{2k+1} x^{2k+1}}{(2k+1)!}$

q - 7

$\frac{\max |f^{n+1}(x)|}{(n+1)!} |x|^{n+1} < 0.1 \qquad x = 3$

$\frac{1}{(n+1)!} |3|^{n+1} < \frac{1}{10}$

$\frac{3^{n+1}}{(n+1)!} < 10$

$10 < \frac{(n+1)!}{3^{n+1}}$

Q-9 $\quad f(x) = e^{3x} \qquad n = 6$

$f'(x) = 3e^{3x}$

$f''(x) = 3^2 e^{3x}$

$\vdots$

$f^{6}(x) = 3^6 e^{3x}$

$f^{7}(x) = 3^7 e^{3x}$

$$\frac{\max |f^{n+1}(c)|}{(n+1)!} |x|^{n+1}$$

$n = 6$

$n+1 = 7$

$$\frac{3^7 e^{3c} \cdot x^7}{7!} = \frac{243 x^7 e^{3c}}{560}$$

Q-10 $\quad g(x) = 2x^{-1}$

Take derival $\qquad$ Eval at point $\qquad (x-1)$ Terms

$g(x) = 2x^{-1} \qquad g(1) = 2(1) \qquad \frac{2}{0!}(x-1)^0$

$g'(x) = -2x^{-2} \qquad g'(1) = -2(1!) \qquad \frac{-2 \cdot 1!}{1!}(x-1)^1$

$g''(x) = -2(-2)x^{-3} \qquad g''(1) = 2(2!) \qquad \frac{2(\cancel{2!})(x-1)^2}{\cancel{2!}}$

$g'''(x) = 2(1 \cdot 2 \cdot -3) \qquad g'''(1) = -2(3!) \qquad \frac{-2(\cancel{3!})(x-1)^3}{\cancel{3!}}$

$$\sum (2)(-1)^k (x-1)^k$$

$$\frac{(x-c)^n}{n!}$$

Midterm B

9-12

$$\int_0^{1/2} \frac{2x^2}{(1-x^2)^{3/2}}\,dx$$

$x = \sin\theta$

$dx = \cos\theta\, d\theta$

$$\int \frac{2x^2}{(1-x^2)^{3/2}}\,dx$$

$$= \int \frac{2\sin^2\theta \cos\theta\, d\theta}{(1-\sin^2\theta)^{3/2}}$$

$$= \int \frac{2\sin^2\theta \cancel{\cos\theta}\, d\theta}{\cos^{\cancel{3}\,2}\theta} = \int 2\tan^2\theta\, d\theta$$

$$= 2\int (\sec^2\theta - 1)\, d\theta$$

$$= 2\tan\theta - 2\theta + C$$

$$= 2\frac{x}{\sqrt{1-x^2}} - 2\sin^{-1}x + C$$

$$\int_0^{1/2} \frac{2x^2}{(\sqrt{1-x^2})^{3/2}}\,dx = \frac{2x}{\sqrt{1-x^2}} - 2\sin^{-1}x \Big|_0^{1/2}$$

$$= \left(\frac{1}{\sqrt{1-\frac{1}{4}}} - 2\sin^{-1}\frac{1}{2}\right)$$

$$= \frac{1}{\sqrt{\frac{3}{4}}} - \cancel{2}\frac{\pi}{6/3} = \frac{2}{\sqrt{3}} - \frac{\pi}{3}$$

$$= \frac{2\sqrt{3}}{3} - \frac{\pi}{3}$$

9-18

$$\frac{(b-a)^3}{12n^2}M \qquad |f''(c)| \le M$$

$$\int_0^4 \cos 2x\,dx \qquad \varepsilon = 0.02 \qquad f(x) = \cos 2x$$

$$f''(x) = -4\cos 2x$$

$15^2 \ldots 95^2$

$$\frac{(4-0)^3}{12n^2}4 \le 0.02 \qquad |f''(x)| \le 4$$

$35^2 = 1225$

$$\frac{4^4}{12 \cdot n^2} \le \frac{2}{100}$$

$$\begin{array}{r} 35 \\ 35 \\ +135 \\ \hline 1225 \end{array} \qquad \begin{array}{r} 25 \\ +125 \\ \hline 625 \end{array}$$

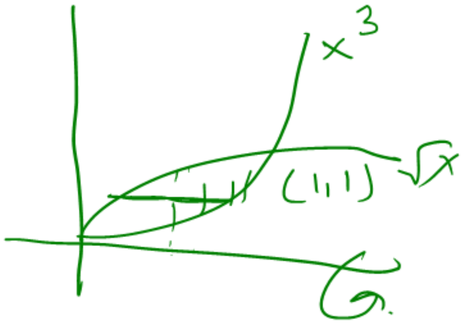
$$\frac{4^4 \cdot 100}{12 \cdot 2} \le n^2$$

$30^2 = 900$
$33^2 =$
$34^2 =$
$35^2 = 1225$

$$1066.67 \le n^2 \qquad n = 33$$

(*) $y=\sqrt{x}$ $\quad y=x^3$

Area $=\int_0^1 (\sqrt{x}-x^3)\,dx$



Rotated

x-axis

$$V=\pi\int_0^1 (\sqrt{x})^2-(x^3)^2\,dx \quad (W)$$

$$=2\pi\int_0^1 y\,(y^{1/3}-y^2)\,dy \quad (S)$$

$y=x^3$

$y^{1/3}$

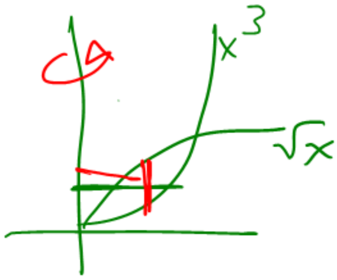
$y=\sqrt{x}$

$y^2=x$

Y-axis

$$V=\pi\int_0^1 \left((y^{1/3})^2-(y^2)^2\right)dy \quad (W)$$

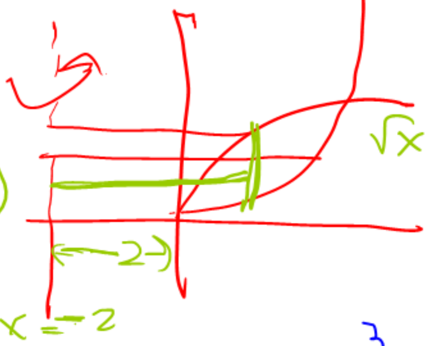
$$V=2\pi\int_0^1 x\,(\sqrt{x}-x^3)\,dx \quad (S)$$



x = −2

$$V=\pi\int_0^1 \left((2+y^{1/3})^2-(2+y^2)^2\right)dy \quad (W)$$

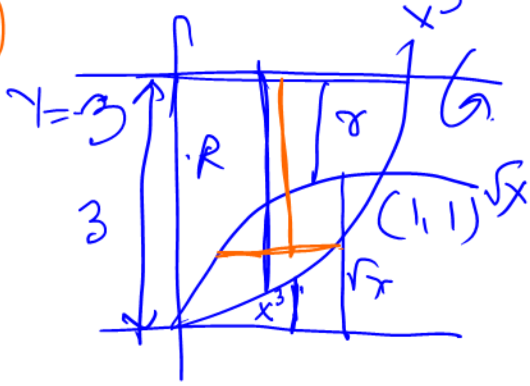
$$=2\pi\int_0^1 (2+x)(\sqrt{x}-x^3)\,dx \quad (S)$$



Y = 3

$$V=\pi\int_0^1 \left((3-x^3)^2-(3-\sqrt{x})^2\right)dx \quad W$$

$$=2\pi\int_0^1 (3-y)(y^{1/3}-y^2)\,dy \quad S$$



Sm of 98

$$\sin x = \sum \frac{(-1)^n x^{2n+1}}{(2n+1)!}$$

$$\sin x^2 = \sum \frac{(-1)^n (x^2)^{2n+1}}{(2n+1)!}$$

$$= \sum \frac{(-1)^n x^{4n+2}}{(2n+1)!}$$

5

$$\frac{\max |f^{n+1}(a)|}{(n+1)!} |x|^{n+1}$$

$f(x) = e^x$

$f'(x) = e^x$

$f^n(x) = e^x$

$|f^n(x)| \le e^x$

$x \in [0,1]$

$|f^n(x)| = e < 3$

$$\frac{3}{(n+1)!} \le \frac{1}{1000}$$

$$3000 \le (n+1)!$$

$6! = 720$

$7! = 5040$

Change I took Error as 0.001 (as I saw from the student) but I spoke with the instructor Error is 0.0001

so $\frac{3}{(n+1)!} \le \frac{1}{10000} \Rightarrow 30000 \le (n+1)!$

$7! = 5040$ $8! = 40320$

$n+1 = 8$

$n = 7$