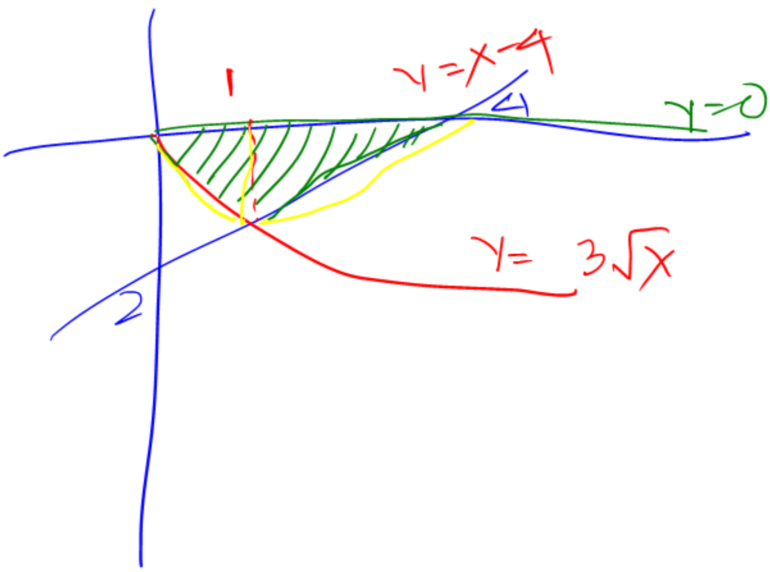
Midterm
q-4



$$\int_0^1 3\sqrt{x}\,dx + \int^4 (4-x)\,dx$$

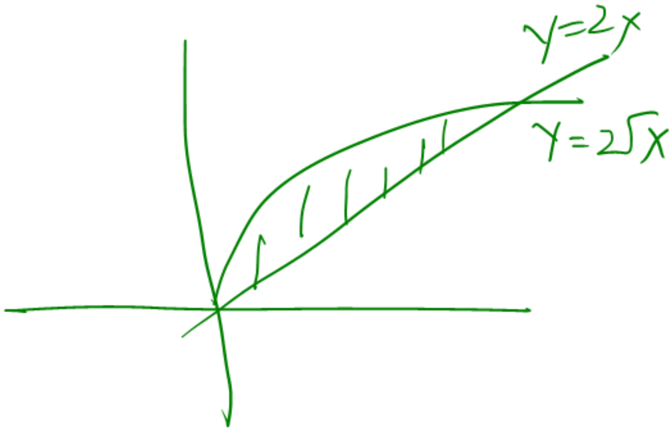
$y=2\sqrt{x}$ $\qquad$ $y=2x$

$$\text{Area}=\int_0^1 2\sqrt{x} - 2x\,dx$$

$$= 2\frac{x^{3/2}}{3/2} - \frac{\cancel{2}x^2}{\cancel{2}}\Big|_0^1$$

$$= \frac{4}{3}x^{3/2} - x^2\Big|_0^1$$

$$= \frac{4}{3} - 1 = \frac{1}{3}$$



$\cancel{2}\sqrt{x} = \cancel{2}x$

$x = x^2$

$x^2 - x = 0$

$x(x-1)=0$

$x = 0, 1$

(*) Avg value $= \frac{1}{2}\int_0^2 (2x - x^2)\,dx$

$$= \frac{1}{2}\left[x^2 - \frac{x^3}{3}\right]_0^2$$

$$= \frac{1}{2}\left[4 - \frac{8}{3}\right] = \frac{1}{2}\left[\frac{12-8}{3}\right]$$

$$= \frac{1}{2}\,\frac{4}{3} = \frac{2}{3}$$

$$\frac{2}{3} = 2c - c^2$$

$$2 = 6c - 3\cancel{x}c^2$$

$$3c^2 - 6x + 2 = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-6) \pm \sqrt{36 - 4\cdot 3 \cdot +2}}{2\cdot 3}$$

$$= \frac{6 \pm \sqrt{36 - 24}}{6} = \frac{6 \pm \sqrt{12}}{6}$$

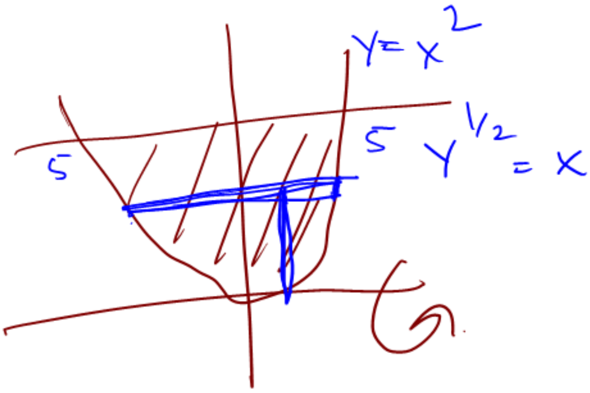
$$= \frac{6}{6} \pm \frac{\sqrt{12}}{6}$$

$$= 1 \pm \frac{\sqrt{4\cdot 3}}{6}$$

$$= 1 \pm \frac{\cancel{2}\sqrt{3}}{\cancel{6}3} = 1 \pm \frac{\sqrt{3}}{3}$$

Midterm A

Q-10



$$\int_0^5 2\pi y\left(y^{\frac12} - (-y^{1/2})\right) dy$$

$$= \int_0^5 2\pi y \, 2y^{1/2}\, dy$$

$$= \int_0^5 4\pi y^{3/2}\, dy$$

Q-13

(*) Q-10

$$\int \frac{e^x \, dx}{e^{2x} + 9e^x + 20}$$

$u = e^x$

$du = e^x dx$

$$= \int \frac{du}{u^2 + 9u + 20}$$

$u^2 + 9u + 20$

$= u^2 + 4u + 5u + 20$

$= (u+4)(u+5)$

$$= \int \frac{du}{(u+4)(u+5)}$$

$$\frac{1}{(u+4)(u+5)} = \frac{A}{u+4} + \frac{B}{u+5}$$

$$1 = A(u+5) + B(u+4)$$

$u = -5 \quad 1 = -B$

$u = -4 \quad 1 = A$

$$= \int \left(\frac{1}{u+4} - \frac{1}{u+5}\right) du = \ln|u+4| - \ln|u+5| + C$$

$$= \ln\left|\frac{u+4}{u+5}\right| + C$$

$$= \ln\left|\frac{e^x + 4}{e^x + 5}\right| + C$$

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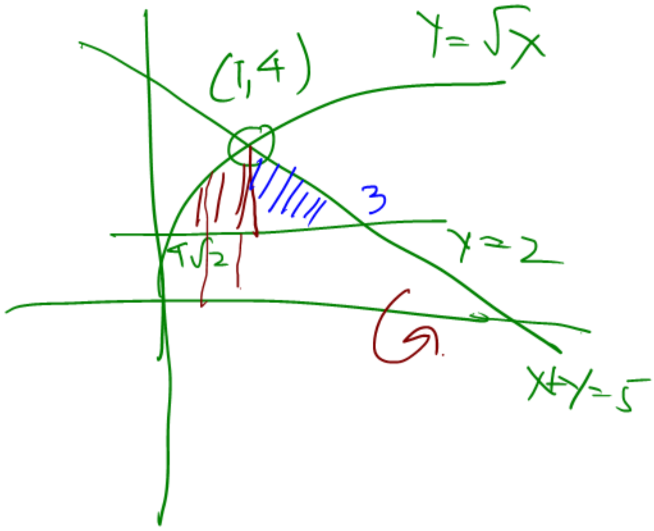
(*) $y = 4\sqrt{x}$ $\quad x+y=5$ $\quad y=2$

$$\pi\int_0^1 (\sqrt{x})^2 - 2^2\,dx$$

$$\pi\int_1^3 (5-x)^2 - 2^2\,dx$$



midterm 13
9-9

(*) $\int \tan^4 2x\,dx$

$= \int \tan^2 2x \tan^2 2x\,dx$

$= \int (\sec^2 2x - 1)\tan^2 2x\,dx$

$= \int \sec^2 2x \tan^2 2x\,dx - \int \tan^2 2x\,dx$

$= \int \sec^2 2x \tan^2 2x\,dx - \int (\sec^2 2x - 1)\,dx$

$u = \tan 2x$
$du = 2\sec^2 2x\,dx$
$\frac{du}{2} = \sec^2 2x\,dx$

$= \int \sec^2 2x \tan^2 2x\,dx - \int \sec^2 2x\,dx + \int dx$

$= \frac{1}{2}\int u^2\,du - \int \sec^2 2x\,dx + \int dx$

$= \frac{1}{2}\frac{u^3}{3} - \frac{\tan 2x}{2} + x + C$

$= \frac{\tan^3 2x}{6} - \frac{\tan 2x}{2} + x + C$

* Mid term B

9-11

$$\int \frac{4x^2\,dx}{\sqrt{64-x^2}} \qquad x = 8\sin\theta \qquad dx = 8\cos\theta\,d\theta$$

$$= \int \frac{4 \cdot 64 \cdot \sin^2\theta \cdot 8\cos\theta\,d\theta}{\sqrt{64 - 64\sin^2\theta}}$$

$$= \int \frac{4 \cdot 64 \cdot \sin^2\theta \cdot \cancel{8}\cancel{\cos\theta}\,d\theta}{\cancel{8}\cancel{\cos\theta}} \qquad \sin^2\theta = \frac{1}{2}(1-\cos 2\theta)$$

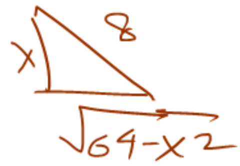
$$= 256 \int \sin^2\theta\,d\theta \qquad x = 8\sin\theta$$

$$= \frac{256}{2} \int (1 - \cos 2\theta)\,d\theta \qquad \frac{x}{8} = \sin\theta$$

$$= 128\left[\theta - \frac{\sin 2\theta}{2}\right] + C \qquad \sin^{-1}\frac{x}{8} = \theta$$

$$= 128\left[\theta - \frac{\cancel{2}\sin\theta\cos\theta}{\cancel{2}}\right] + C \qquad \sin 2\theta = 2\sin\theta\cos\theta$$

$$= 128\left[\sin^{-1}\frac{x}{8} - \frac{x}{8}\,\frac{\sqrt{64-x^2}}{8}\right] + C \qquad x = 8\sin\theta$$



$$= 128\sin^{-1}\frac{x}{8} - \frac{128}{64}\,x\sqrt{64-x^2} + C$$

$$= 128\sin^{-1}\frac{x}{8} - 2x\sqrt{64-x^2} + C$$

Midterm 3

4-5

$$\int \frac{2x^2}{\sqrt{8+x^2}}\,dx \qquad x = \sqrt{8}\tan\theta \qquad dx = \sqrt{8}\sec^2\theta\, d\theta$$

$$= \int \frac{2\cdot 8\tan^2\theta\cdot\sqrt{8}\sec^2\theta\, d\theta}{\sqrt{8+8\tan^2\theta}}$$

$$= \int \frac{2\cdot 8\tan^2\theta\,\cancel{\sqrt{8}}\,\sec^{\cancel{2}}\theta\, d\theta}{\cancel{\sqrt{8}}\,\cancel{\sec}\theta}$$

$$= 16\int \tan^2\theta\,\sec\theta\, d\theta$$

$$= 16\int (\sec^2\theta - 1)\sec\theta\, d\theta$$

$$= 16\int (\sec^3\theta - \sec\theta)\, d\theta$$

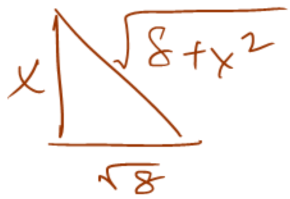
$$= 16\left[\left(\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right) - \ln|\sec\theta+\tan\theta|\right] + C$$

$$= 16\left[\frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta|\right] + C$$

$$= 8\sec\theta\tan\theta - 8\ln|\sec\theta+\tan\theta| + C$$

$$= \cancel{8}\,\frac{\sqrt{8+x^2}}{\cancel{\sqrt{8}}}\,\frac{x}{\cancel{\sqrt{8}}} - 8\ln\left|\frac{\sqrt{8+x^2}}{\sqrt{8}} + \frac{x}{\sqrt{8}}\right| + C \qquad x = \sqrt{8}\tan\theta$$

$$= x\sqrt{8+x^2} - 8\ln\left|\frac{x+\sqrt{8+x^2}}{\sqrt{8}}\right| + C$$



q-6

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$$\int 9 \arcsin 6x\,dx \qquad u = \sin^{-1}6x \qquad dv = dx$$

$$= 9\int \sin^{-1}6x\,dx \qquad du = \frac{6\,dx}{\sqrt{1-36x^2}} \qquad v = x$$

$$= 9\left[x\sin^{-1}x - \int \frac{x\,6\,dx}{\sqrt{1-36x^2}}\right]$$

$$u = 1-36x^2$$

$$du = -72x\,dx$$

$$\frac{-du}{72} = x\,dx$$

$$= 9x\sin^{-1}x + \frac{\overset{3}{\cancel{6}\cdot\cancel{9}}}{\underset{4}{\cancel{72}\,\cancel{12}}}\int \frac{du}{\sqrt{u}}$$

$$= 9x\sin^{-1}x + \frac{3}{4}\,\frac{u^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} + C$$

$$= 9x\sin^{-1}x + \frac{3}{\cancel{4}\,2}\,\frac{u^{1/2}}{\cancel{1/2}} + C$$

$$= 9x\sin^{-1}x + \frac{3}{2}\sqrt{1-36x^2} + C$$

Q-7 $\int \frac{10}{(x-6)(x+4)}\,dx$

$$\frac{10}{(x-6)(x+4)} = \frac{A}{x-6} + \frac{B}{x+4}$$

$$10 = A(x+4) + B(x-6)$$

$x=6$ $\quad 10 = 10A \Rightarrow A = 1$

$x=-4$ $\quad 10 = -10B \Rightarrow B = -1$

$$\int \frac{1}{x-6}\,dx - \int \frac{1}{x+4}\,dx$$

$$= \ln|x-6| - \ln|x+4| + C$$

$$= \ln\left|\frac{x-6}{x+4}\right| + C$$

(*) $\int_0^{\pi/2} 8x^2 \sin x\,dx = 8\int_0^{\pi/2} x^2 \sin x\,dx$

$\int x^2 \sin x\,dx$ $\qquad u = x^2 \qquad dv = \sin x\,dx$

$\qquad du = 2x\,dx \qquad v = -\cos x$

$= -x^2\cos x + \int 2x\cos x\,dx$ $\qquad u = x \qquad dv = \cos x\,dx$

$= -x^2\cos x + 2x\sin x - 2\int \sin x\,dx$ $\qquad du = dx \qquad v = \sin x$

$= -x^2\cos x + 2x\sin x + 2\cos x + C$

$$\int_0^{\pi/2} 8x^2 \sin x\,dx = 8\left[-x^2\cos x + 2x + 2\cos x\right]_0^{\pi/2}$$

$$= 8\left[\left(-\left(\frac{\pi}{2}\right)^2\cos\frac{\pi}{2} + 2\frac{\pi}{2} + 2\cos\frac{\pi}{2}\right) - (2\cos 0)\right]$$

$$= 8\left[\pi - 2\right] = 8\pi - 16$$

(4) $f(x) = \sqrt{e^{(1.2)}}$ $\quad 0.01$

$= e^{\frac{1.2}{2}} = e^{0.6}$

Find n using 0.01

$$\max_{0 \le c \le x} \frac{|f^{n+1}(c)|}{(n+1)!} |x|^{n+1} \le 0.01$$

$n = 3/4/5/$

$f(x) \approx 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!}$

$x = 0.6$