Q-22

#4 $(x-4)^2+y^2=16$

$x=r\cos\theta \qquad y=r\sin\theta$

$$x^2+y^2=r^2\cos^2\theta+r^2\sin^2\theta$$
$$=r^2(\cos^2\theta+\sin^2\theta)=r^2$$
$$x^2+y^2=r^2$$

$$(x-4)^2+y^2=16$$
$$x^2+16-8y+y^2=16$$
$$x^2+y^2=8y$$
$$r^2=8r\sin\theta$$
$$r=8\sin\theta$$

#9 $r=2\cos(3\theta)$ (3: odd)

3-petals

$r=2\cos(4\theta)$ (4: even)

8-petals

$\theta\in[0,\pi/2]$

$2\cos(3\pi/6)=2\cos(\frac{\pi}{2})$

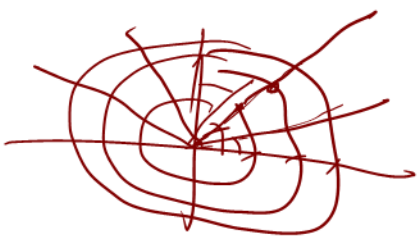
$2\cos(\frac{3\pi}{3})=2\cos(\pi)$

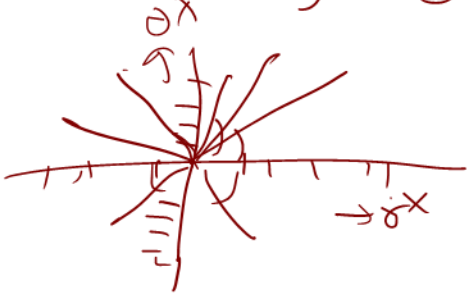
$2\cos(\frac{3\pi}{2})=0$

θ	0	$\pi/6$	$\pi/3$	$\pi/2$
r	2	0	-2	0

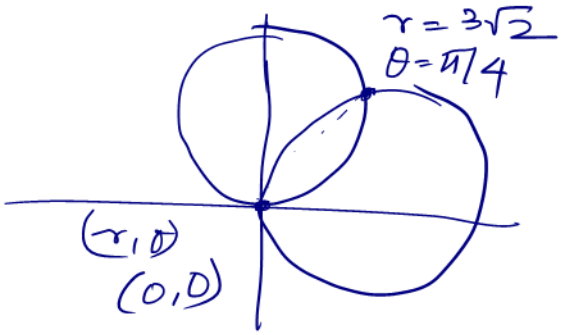
r,θ

$(0,2)$





#10 $r = 6\cos\theta$ $r = 6\sin\theta$



$6\cos\theta = 6\sin\theta$

$1 = \tan\theta$

$\theta = \pi/4$

$\theta = \pi/4 \quad r = 6\cos\pi/4$

$= 6\frac{\sqrt{2}}{2} = 3\sqrt{2}$

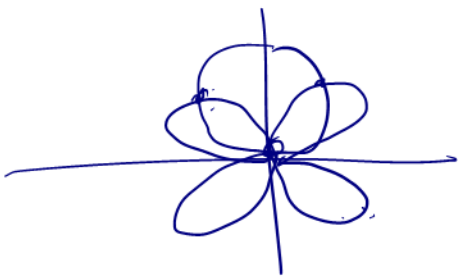
$x = r\cos\theta \qquad y = r\sin\theta)$

$(r,\theta) \equiv (0,0) \quad (x,y) = (0\cos 0,\ 0\sin\theta) = (0,0)$

$(r,\theta) \equiv (3\sqrt{2}, \pi/4) \quad (x,y) = \left(3\sqrt{2}\cos\frac{\pi}{4},\ 3\sqrt{2}\sin\frac{\pi}{4}\right)$

$= \left(\frac{3\sqrt{2}\sqrt{2}}{2},\ \frac{3\sqrt{2}\sqrt{2}}{2}\right) = (3,3)$

(*) $r = \sin\theta \quad r = \sin 2\theta$



$\sin\theta = \sin 2\theta$

$\sin\theta = 2\sin\theta\cos\theta$

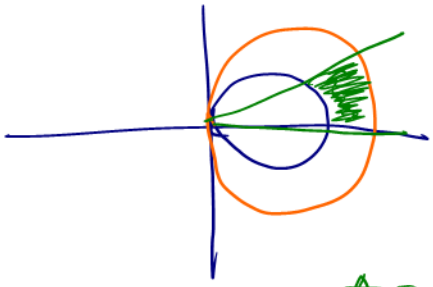
$2\sin\theta\cos\theta - \sin\theta = 0$

$\sin\theta(2\cos\theta - 1) = 0$

$\theta = 0 \qquad \cos = \frac{1}{2}$

Q-23
#3

$r = 4\cos\theta \quad r = 2\cos\theta \quad \theta = 0 \quad \theta = \pi/4$



$$A = \frac{1}{2}\int r^2 d\theta$$

$$A = \frac{1}{2}\int_0^{\pi/4} \left((4\cos\theta)^2 - (2\cos\theta)^2\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} 16\cos^2\theta - 4\cos^2\theta \, d\theta$$

$$= \frac{1}{2}\int_0^{\pi/4} 6 + 6\cos^2\theta \, d\theta$$

$$= 6\int_0^{\pi/4} \cos^2\theta \, d\theta$$

$$= 6\int_0^{\pi/4} \frac{1}{2}(1+\cos 2\theta)\, d\theta$$

$$= 3\int_0^{\pi/4} (1+\cos 2\theta)\, d\theta$$

$$= 3\left[\theta + \frac{\sin 2\theta}{2}\right]_0^{\pi/4}$$

$$= 3\left[\pi/4 + \frac{\sin \pi/2}{2}\right] = \frac{3\pi}{4} + \frac{3}{2}$$

Q-23
#10

$r = 3\sec\theta \qquad \theta \in [0, \pi/3]$

$r' = 3\sec\theta\tan\theta$

$$L = \int_{\alpha}^{\beta} \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta$$

$$= \int_{0}^{\pi/3} \sqrt{9\sec^2\theta + 9\sec^2\theta\tan^2\theta}\, d\theta$$

$$= \int_{0}^{\pi/3} \sqrt{9\sec^2\theta(1+\tan^2\theta)}\, d\theta$$

$$= \int_{0}^{\pi/3} \sqrt{9\sec^4\theta}\, d\theta$$

$$= \int_{0}^{\pi/3} 3\sec^2\theta\, d\theta = 3\tan\theta\Big|_0^{\pi/3}$$

$\sqrt{3}$, 2, 1, 2

$$= 3\tan\frac{\pi}{3} - 3\tan 0$$

$$= 3\sqrt{3}$$

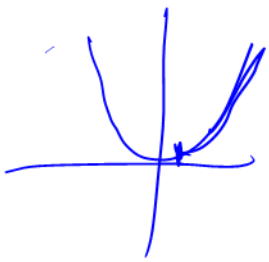
Q-24
#6

$\left(\frac{1}{t}, \frac{2}{t^2}\right)$

$x = \frac{1}{t} \qquad y = \frac{2}{t^2}$

$y = 2\left(\frac{1}{t}\right)^2$

$y = 2x^2$



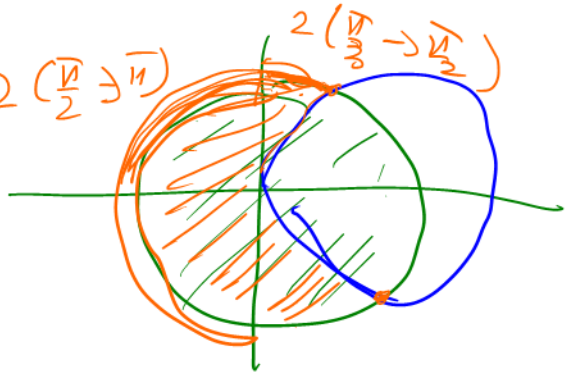
$t \in (0, 2]$ $\quad x = 2 \quad y = 8$

Q-23
#4

$r = 8$ $\quad r = 16\cos\theta$

$8 = 16\cos\theta$

$\frac{1}{2} = \cos\theta$



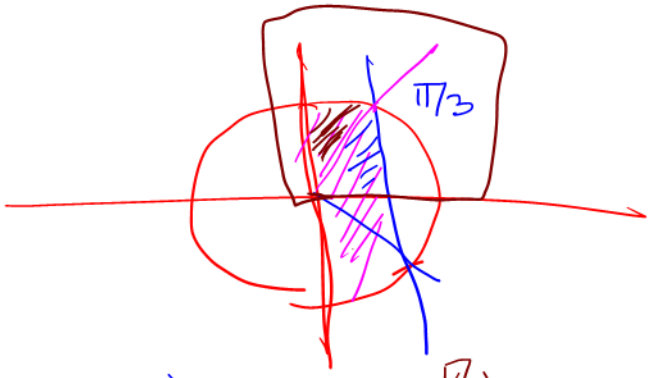
$2\int_{\pi/2}^{\pi} \frac{1}{2} 8^2 \, d\theta$

$2\int_{\pi/3}^{\pi/2} \frac{1}{2}\left(8^2 - (16\cos\theta)^2\right) d\theta$

#5 $\quad r = 14$ $\quad \theta = \pi/2$ $\quad r = 7\sec\theta$

$r\cos\theta = 7$

$x = 7$



$14 = 7\sec\theta$

$2 = \sec\theta$

$\cos\theta = \frac{1}{2}$

$2\int_{0}^{\pi/3} \frac{1}{2}(7\sec\theta)^2 \, d\theta + 2\int_{\pi/3}^{\pi/2} (14)^2 \, d\theta$

Q-24 $x = x(t)$ $y = y(t)$

#10

$$f(x) = x^8 - x^2 + 3$$

$(-8, 4)$ $(-7, -1)$

$x = t$

$$f(t) = t^8 - t^2 + 3$$

$$y(t) = t^8 - t^2 + 3 \quad t \in [-8, -7]$$

Emcf 10.3

#3 $x(t) = 3t^2$ $y(t) = 2t + 1$

$$y - 1 = 2t$$

$$\frac{y-1}{2} = t$$

$$x = 3\left(\frac{y-1}{2}\right)^2$$

$$x = \frac{3(y-1)^2}{4} \Rightarrow 4x = 3y^2 + 3 - 6y$$

#5 $$L = \int_{\alpha}^{\beta} \sqrt{r(\theta)^2 + r'(\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{(1-\cos\theta)^2 + (\sin\theta)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{1 + \cos^2\theta - 2\cos\theta + \sin^2\theta}\, d\theta$$

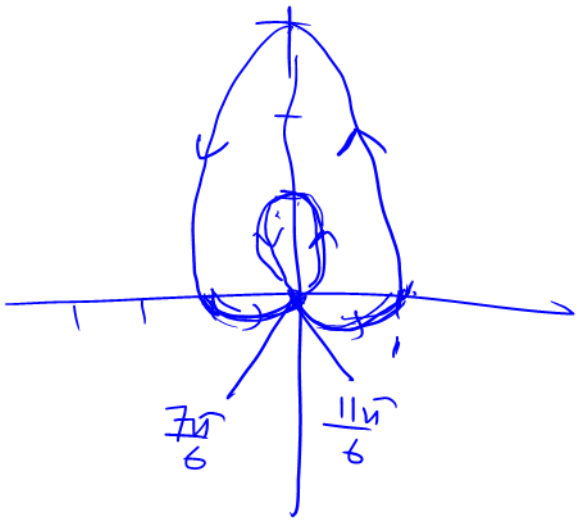
$$= \int_0^{2\pi} \sqrt{2 - 2\cos\theta}\, d\theta$$

Emf 10.2

5

$r = 1 + 2\sin\theta$

θ	r
0	1
$\frac{\pi}{2}$	3
π	1
$\frac{3\pi}{2}$	-1
2π	1



$r = 0$

$1 + 2\sin\theta = 0$

$\sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

$$\int_{\frac{7\pi}{6}}^{11\pi/6} \frac{1}{2}(r(\theta))^2 \, d\theta$$