(*) $\int 2\pi\, y(t)\sqrt{x'(t)^2 + y'(t)^2}\, dt$

$x(t) = t^2 + 5 \qquad y(t) = 5t \qquad t \in [0, 4]$

$x'(t) = 2t \qquad y'(t) = 5$

$$\int_0^4 2\pi\, 5t\sqrt{4t^2 + 25}\, dt$$

$$= 10\pi \int_0^4 t\sqrt{4t^2+25}\, dt$$

$$\int t\sqrt{4t^2+25}\, dt = \frac{1}{8}\int \sqrt{u}\, du \qquad u = 4t^2 + 25$$

$$= \frac{1}{\cancel{8}_{4}}\, \frac{u^{3/2}}{3/\cancel{2}} + C = \frac{(4t^2+25)^{3/2}}{12} + C \qquad \underline{du} = 8t\,dt$$

$$\overset{5}{\cancel{10}}\pi\, \frac{(4t^2+25)^{3/2}}{\overset{6}{\cancel{12}}}\Bigg|_0^4 = \frac{5\pi}{6}\left((64+25)^{3/2} - (25)^{3/2}\right)$$

$$= \frac{5\pi}{6}\left((89)^{3/2} - (25)^{3/2}\right)$$

$$= \frac{5\pi}{6}\left(89\sqrt{89} - 125\right)$$

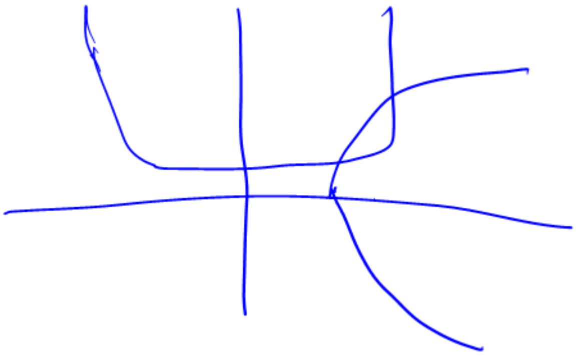
$$= \pi\left(\frac{445}{6}\sqrt{89} - \frac{625}{6}\right)$$

Prac Final

#3

$y = 4\sqrt{x} + 2$

$y = \frac{1}{16}x^2 + 2$



$4\sqrt{x} + \cancel{2} = \frac{1}{16}x^2 + \cancel{2}$

$64\sqrt{x} = x^2$

$64^2 x = x^4$

$x(x^3 - 64^2) = 0$

$x = 0 \qquad x^3 = 64^2$

$x = (64)^{2/3} = (4^3)^{2/3} = 16$

$\int_0^{16} (4\sqrt{x} + 2) - (\frac{1}{16}x^2 + 2)\, dx$

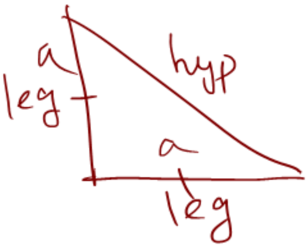
#4 $x = \frac{7y^3}{2} \qquad x = \frac{7}{2}$

$\int_0^{7/2} 2\pi x \left(\frac{2}{7}x^{1/3}\right) dx$

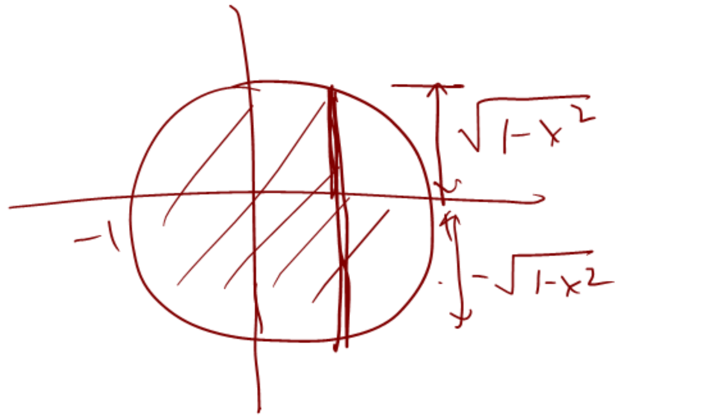
$\pi \int_0^1 \left(\frac{7}{2}\right)^2 - \left(\frac{7y^3}{2}\right)^2 dy$

$\frac{7}{2} = \frac{7}{2}y^3 \Rightarrow y = 1$

5 $x^2+y^2=1$



$A=\frac{1}{2}a^2$



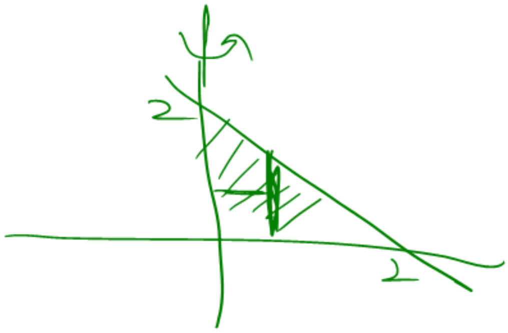
$a = 2\sqrt{1-x^2}$

$A = \frac{1}{2}\left(2\sqrt{1-x^2}\right)^2$

$V=\int_{-1}^{1}\frac{1}{2}\left(2\sqrt{1-x^2}\right)^2dx$

OR $=\int_{0}^{1}2\cdot\frac{1}{2}\left(2\sqrt{1-x^2}\right)^2dx$

6 $x+y=2$ $x=0,\ y=0$



$\int_{0}^{2}2\pi x(2-x)\,dx$

10 $-6y'+18x^2=0$

$-6\frac{dy}{dx}+18x^2=0$

$+\cancel{6}\,dy = +\cancel{18}^{3}x^2\,dx$

$\int dy = \int 3x^2\,dx$

$y = x^3 + C$

#11

$$k\,t_{1/2} = \ln\left(\tfrac{1}{2}\right)$$

$$k = \frac{1}{1620}\ln\left(\tfrac{1}{2}\right)$$

$$A = A_0\, e^{\frac{1}{\cancel{1620}\ 81}\ln\left(\frac{1}{2}\right)\cancel{800}\ 40}$$

$$= A_0\, e^{\ln\left(\frac{1}{2}\right)\frac{40}{81}}$$

$$= A_0\left(\tfrac{1}{2}\right)^{\frac{40}{81}}$$

$$\approx 71\%$$