Practice Final

#21

$$a_n = \frac{n^{2}}{\sqrt{6n^{4}+1}} = \frac{1}{\sqrt{6}} = \frac{\sqrt{6}}{6}$$

#23 $\sum a_k = \sum \frac{6k-1}{\sqrt{k^6+4}}$

$$\sum b_k = \sum \frac{1}{k^2}$$

$$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{6k^3 - k^2}{\sqrt{k^6+4}} = 6 > 0$$

$\sum b_k$ is conv by p-series test as $p=2$.

$\therefore$ $\sum a_k$ is conv by limit comp test.

#13 $$\int_{1/3}^{3} \frac{6}{\sqrt[3]{3x-1}}\,dx = \lim_{a\to\frac{1}{3}^+} \int_a^3 \frac{6\,dx}{\sqrt[3]{3x-1}}$$

$$\int \frac{6\,dx}{\sqrt[3]{3x-1}}$$

$u = 3x-1$

$du = 3dx$

$$= \frac{1}{3}\int \frac{6\,du}{u^{1/3}}$$

$$= 2\int u^{-1/3}\,du$$

$$= 2\,\frac{u^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} + C$$

$$= \cancel{2}\,\frac{(3x-1)^{2/3}}{\cancel{2}/3} + C = 3(3x-1)^{2/3}$$

$$= \lim_{a\to 1/3^+} 3(3x-1)^{2/3}\Big|_a^3$$

$$= \lim_{a\to 1/3^+} 3\left[(9-1)^{2/3} - (3a-1)^{2/3}\right]$$

$$= 3(8)^{2/3}$$

$$= 3(4) = 12$$

Labquiz 25

#

$x = r\cos\theta$ $\qquad$ $y = r\sin\theta$

$$\frac{dx}{d\theta} = \frac{dr}{d\theta}\cos\theta - r\sin\theta \qquad \frac{dy}{d\theta} = \frac{dr}{d\theta}\sin\theta + r\cos\theta$$

$$\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta}$$

$r = 20 - 10\sin\theta$ $\qquad$ $\theta = 0$

at $\theta = 0$ $\quad r = 20$

$$\frac{dr}{d\theta} = -10\cos\theta \qquad \left.\frac{dr}{d\theta}\right|_{\theta=0} = -10$$

$$\frac{dy}{dx} = \frac{-10\sin\cancel{0} + 20\cos 0}{-10\cos 0 - 20\sin\cancel{0}} = \frac{20}{-10} = -2$$

at $\theta = 0$ $\quad x = 20\cos 0$ $\qquad y = 20\sin 0$

$(20, 0)$

$y - 0 = -2(x - 20)$

$y = -2x + 40$

Practice final

#26

$$\sum \frac{4^k x^{2k}}{(k+2)^2}$$

$$\lim_{k\to\infty} k^{1/k} = 1$$

$$\lim_{k\to\infty} \left| \left(\frac{4^k x^{2k}}{(k+2)^2} \right)^{1/k} \right| = \lim_{k\to\infty} \left| \frac{4x^2}{(k+2)^{2/k}} \right|$$

$$= |4x^2|$$

$x^2 = \frac{1}{4}$

$x = \pm \frac{1}{2}$

$$|4x^2| < 1 \Rightarrow -1 < 4x^2 < 1$$

$$\Rightarrow \cancel{-\frac{1}{4}} < x^2 < \frac{1}{4} \Rightarrow x^2 < \frac{1}{4}$$

$$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

$$x = -\frac{1}{2} \quad \sum \frac{4^k \left(-\frac{1}{2}\right)^{2k}}{(k+2)^2} = \sum \frac{\cancel{4^k} \frac{1}{\cancel{4^k}}}{(k+2)^2}$$

$$= \sum \frac{1}{(k+2)^2} < \sum \frac{1}{k^2}$$

BCT

conv by p-series test $p = 2$ ✓

$$x = \frac{1}{2} \quad \sum \frac{4^k \left(\frac{1}{2}\right)^{2k}}{(k+2)^2} = \sum \frac{1}{(k+2)^2}$$

$$\left[-\frac{1}{2}, \frac{1}{2}\right]$$

Practice Final

#24 $\sum k\left(\frac{1}{4}\right)^k$

$$\lim_{k\to\infty}\left(k\left(\frac{1}{4}\right)^k\right)^{1/k} = \lim_{k\to\infty} k^{1/k}\left(\frac{1}{4}\right) = \frac{1}{4} < 1$$

$\therefore$ By Root test it's convergent

#25 $\sum\left|\frac{6(-1)^k k}{6^k}\right| = \sum\frac{6k}{6^k} = 6\sum\frac{k}{6^k}$

$$\lim_{k\to\infty}\left(\frac{k}{6^k}\right)^{1/k} = \lim_{k\to\infty}\frac{k^{1/k}}{6} = \frac{1}{6} < 1$$

So conv by Root test

Hence abs conv