(*) $\sum \frac{3}{7^k}$ or $\sum \left(\frac{3}{7}\right)^k$ $\quad |r| = \frac{3}{7} < 1$

$= 3\sum \left(\frac{1}{7}\right)^k$

$a_k = \frac{3}{7^k}$ $\quad |r| = \frac{1}{7} < 1$

$$\lim_{k\to\infty} \left(\frac{3}{7^k}\right)^{1/k} = \lim_{k\to\infty} \frac{3^{1/k}}{7} = \frac{1}{7} < 1$$

$\therefore$ Convg by Root test

(*) $\sum \left| \frac{(-1)^n n^4 + 2n^2 - 3}{5n^5 + 3n^3 + 2n^2} \right| = \sum \frac{n^4 + 2n^2 - 3}{5n^5 + 3n^3 + 2n^2}$

$\sum a_n = \sum \frac{n^4 + 2n^2 - 3}{5n^5 + 3n^3 + 2n^2}$ $\qquad \sum b_n = \sum \frac{1}{n}$

$$\lim_{n\to\infty} \frac{a_n}{b_n} = \lim_{n\to\infty} \frac{n^5 + 2n^3 - 3n}{5n^5 + 3n^3 + 2n} = \frac{1}{5} > 0$$

$\sum \frac{1}{n}$ is a divergent as $p = 1$

$\therefore$ $\sum a_n$ is div by LCT

Hence $\sum a_n$ is not abs convg

$a_n = \frac{n^4 + 2n^2 - 3}{5n^5 + 3n^3 + 2n}$
(i) mono dec
(ii) continuous
(iii) $\lim_{n\to\infty} a_n = \lim_{n\to\infty} \frac{n^4 + 2n^2 - 3}{5n^5 + 3n^3 + 2n} = 0$

$\therefore$ $\sum a_n$ is convg by AST

$\therefore$ $\sum a_n$ is cond. convg

Practice final

#24 $\sum k\left(\frac{8}{9}\right)^k$

$$\lim_{k\to\infty}\left(k\left(\frac{8}{9}\right)^k\right)^{1/k} = \lim_{k\to\infty} k^{1/k}\left(\frac{8}{9}\right) = \frac{8}{9} < 1$$

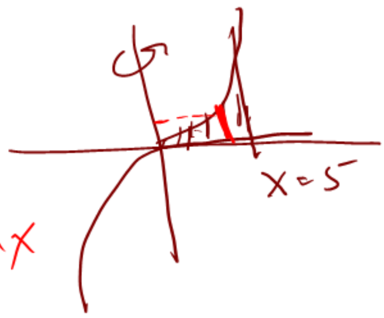
∴ Conv by Root test

#24 $\sum 2ke^{-k^2} = 2\sum ke^{-k^2}$

$$\lim_{k\to\infty}\left(ke^{-k^2}\right)^{1/k} = \lim_{k\to\infty} k^{1/k}e^{-k} = \lim_{k\to\infty}\frac{k^{1/k}}{e^k} = \frac{1}{\infty} = 0 < 1$$

Conv by Root test

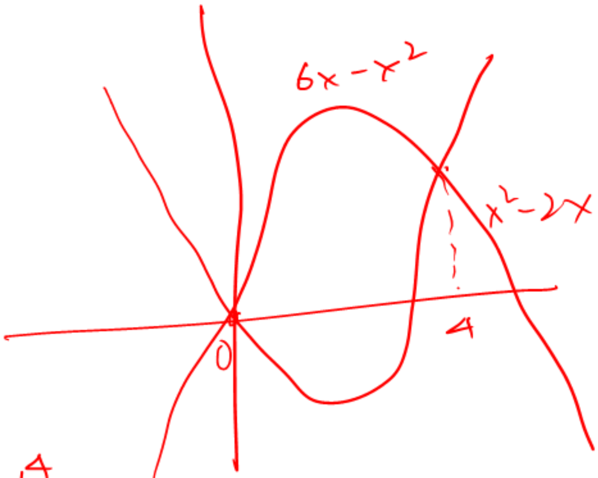
#4 $x = \frac{5y^3}{8}$ $x=5$ $y=0$



$$\int_0^5 2\pi x\left(\frac{8x}{5}\right)^{1/3} dx = 2\pi\int_0^5\left(\frac{8}{5}\right)^{1/3} x^{4/3}\,dx$$

#7 $y = x^2 - 2x$

$y = 6x - x^2$



$$A = \int_0^4 (6x - x^2) - (x^2 - 2x)\, dx$$

$$= \int_0^4 6x - x^2 - x^2 + 2x \, dx = \int_0^4 8x - 2x^2 \, dx$$

#22 $$\sum_0^\infty \frac{2^{k+3}}{3^k} = \sum_0^\infty \frac{2^k \cdot 2^3}{3^k} = 8 \sum_0^\infty \left(\frac{2}{3}\right)^k$$

$r = \frac{2}{3}$ $\quad |r| = \frac{2}{3} < 1$ conv

$$\text{Sum} = \frac{a}{1-r} = \frac{8}{1 - \frac{2}{3}} = \frac{8}{1/3} = 24$$

\# $$\int \tan^2 x \sec x \, dx$$

$$= \int (\sec^2 x - 1) \sec x \, dx$$

$$= \int (\sec^3 x - \sec x) \, dx$$

$$= \int \sec^3 x \, dx - \int \sec x \, dx$$

$$= \frac{1}{2} \left(\sec x \tan x + \ln|\sec x + \tan x| \right) \leftarrow \ln|\sec x + \tan x| + C$$

$$= \frac{1}{2} \sec x \tan x - \frac{1}{2} \ln|\sec x + \tan x| + C$$

$\int \sec^3 x \, dx$ $\qquad u = \sec x \quad dv = \sec^2 x \, dx$

$= \int \sec x \sec^2 x \, dx$ $\qquad du = \sec x \tan x \, dx \quad v = \tan x$

$= \sec x \tan x - \int \sec x \tan x \tan x \, dx$

$= \sec x \tan x - \int \sec \tan^2 x \, dx$

$= \sec x \tan x - \int \sec x (\sec^2 x - 1) \, dx$

$= \sec x \tan x - \int (\sec^3 x - \sec x) dx$

$= \sec x \tan x - \int \sec^3 x + \int \sec x \, dx$

$2 \int \sec^3 x \, dx = \sec x \tan x + \ln|\sec x + \tan x| + C$

$\int \sec^3 x \, dx = \frac{1}{2}(\sec x \tan x + \ln|\sec x + \tan x|) + C$

(*) $\qquad x = t^2 \qquad y = 4^t + 1$

$y - 1 = 4^t$

$\ln(y-1) = \ln 4^t = t \ln(4)$

$\frac{\ln(y-1)}{\ln(4)} = t$

$x = \left(\frac{\ln(y-1)}{\ln 4}\right)^2$

$x = t^2 \qquad y = 4t + 1$

$\frac{y-1}{4} = t$

$x = \left(\frac{y-1}{4}\right)^2$

$16x = (y-1)^2$

$$E = \frac{\max|f^{n+1}(c)|}{(n+1)!}|x|^{n+1} < \frac{1}{(n+1)!}5^{n+1}$$

$$\frac{5^{n+1}}{(n+1)!} \leq \frac{1}{10}$$

$$10 \leq \frac{(n+1)!}{5^{n+1}}$$

$n=1$ $\quad \frac{2!}{5^2} = \frac{2}{25}$ $\quad n+1=13 \quad 5.1$

$n=2$ $\quad \frac{3!}{5^3} = \frac{6}{125}$ $\quad n+1=14 \quad 14.2$

$n=5$ $\quad \frac{6!}{5^7} = 720$ $\quad n=13$

$$\# \sum_{0}^{\infty} \frac{1-6^k}{7^k} = \sum_{0}^{\infty}\left[\left(\frac{1}{7}\right)^k - \left(\frac{6}{7}\right)^k\right]$$

$$\frac{a}{1-r} \qquad = \frac{1}{1-\frac{1}{7}} - \frac{1}{1-\frac{6}{7}}$$

$$= \frac{1}{\frac{6}{7}} - \frac{1}{\frac{1}{7}}$$

$$= \frac{7}{6} - 7 = \frac{7-42}{6} = -\frac{35}{6}$$