Lab quiz 13

1. The polar form of the ewuation $x^2 + y^2 + 2x + 6y = 0$ is:

   (A) $r = \sqrt{10}$

   (B) $r = 3sec\theta + 6tan\theta$

   (C) $r = -2sin\theta - 6cos\theta$

   (D) $r = -2cos\theta - 6sin\theta$

   (E) $r = -2cos\theta + 6sin\theta$

   Solution:

   The substitution is $x = rcos\theta$ and $y = rsin\theta$.
   So we have $x^2 + y^2 = r^2cos^2\theta + r^2sin^2\theta = r^2(cos^2\theta + sin^2\theta) = r^2$
   Then by substitution we get:
   $r^2 + 2rcos\theta + 6rsin\theta = 0$
   $\Rightarrow r^2 = -2rcos\theta - 6rsin\theta$
   $\Rightarrow r^2 = r(-2cos\theta - 6sin\theta)$
   $\Rightarrow r = -2cos\theta - 6sin\theta$

2. For the parametric curve: $x(t) = 3 + 2cos(t), y(t) = 1 + 4sin(t), t \in [0, 2\pi)$. Give an equation in x and y that represents this curve.

(A) $\frac{x^2}{4} + \frac{y^2}{16} = 1$

(B) $\frac{(x-3)^2}{4} + \frac{(y-1)^2}{16} = 1$

(C) $\frac{(x+3)^2}{16} + \frac{(y+1)^2}{4} = 1$

(D) $\frac{(x+3)^2}{4} + \frac{(y+1)^2}{16} = 1$

(E) $\frac{(x-2)^2}{16} + \frac{(y-4)^2}{4} = 1$

Solution:

We have
$x(t) = 3 + 2cos(t)$ and $y(t) = 1 + 4sin(t)$
$\Rightarrow x - 3 = 2cos(t)$ and $y - 1 = 4sin(t)$
$\Rightarrow \frac{x-3}{2} = cos(t)$ and $\frac{y-1}{4} = sin(t)$
Now
$cos^2(t) + sin^2(t) = 1 \Rightarrow (\frac{x-3}{2})^2 + (\frac{y-1}{4})^2 = 1 \Rightarrow \frac{(x-3)^2}{4} + \frac{(y-1)^2}{16} = 1$

3. For the parametric curve: $x(t) = 3 + 2cos(t), y(t) = 1 + 4sin(t), t \in [0, 2\pi)$. state the points (x, y) where tangent line are horizontal.

(A) (3,5), (3,1)

(B) (5,5) , (5,1)

(C) (3,5), (3,-3)

(D) (5,1), (1,1)

(E) (4,1), (1,1)

Solution:

$x' = -2sin(t)$ and $y' = 4cos(t)$.
For horizontal tangent we need $y' = 0$ and $x' \neq 0$
$y' = 0 \Rightarrow 4cos(t) = 0 \Rightarrow cos(t) = 0 \Rightarrow t = \frac{\pi}{2}, \frac{3\pi}{2}$
Now $x'(\frac{\pi}{2}) = -2 \neq 0$ and $x'(\frac{3\pi}{2}) = 2 \neq 0$
Thus, the horizontal tangents occur at $t = \frac{\pi}{2}, \frac{3\pi}{2}$
Therefore, the points are
$(x(\frac{\pi}{2}), y(\frac{\pi}{2})) = (3, 5)$
and
$(x(\frac{3\pi}{2}), y(\frac{3\pi}{2})) = (3, -3)$

4. For the parametric curve: $x(t) = 3 + 2cos(t), y(t) = 1 + 4sin(t), t \in [0, 2\pi)$. state the points (x, y) where tangent line are vertical.

   (A) (3,5), (3,1)

   (B) (5,5) , (5,1)

   (C) (3,5), (3,-3)

   (D) (5,1), (1,1)

   (E) (4,1), (1,1)

   Solution:

   $x' = -2sin(t)$ and $y' = 4cos(t)$.
   For vertical tangent we need $x' = 0$
   $x' = 0 \Rightarrow -2sin(t) = 0 \Rightarrow sin(t) = 0 \Rightarrow t = 0, \pi$

   $(x(0), y(0)) = (5, 1)$
   and
   $(x(\pi), y(\pi)) = (1, 1)$

   (Notice that $x' = 0$ also for $t = 2\pi, 3\pi$ and so on but as $t \in [0, 2\pi)$ so we do not include 2π and later points)

5. The equation of the tangent line to the curve defined by $F(t) = (t^2 + 1, 2^t)$ at the point $y = 4$ is:

(A) $y - 4 = 4ln(2)(x - 5)$

(B) $y - 4 = 4ln(2)(x - 2)$

(C) $y - 4 = ln(2)(x - 2)$

(D) $y - 4 = 4(x - 5)$

(E) $y - 4 = ln(2)(x - 5)$

Solution:

$x(t) = t^2 + 1$ and $y(t) = 2^t$.
Given $y = 4 \Rightarrow 2^t = 4 \Rightarrow t = 2$
Hence, $x(2) = 5$ and $y(2) = 4 \Rightarrow (x, y) = (5, 4)$
Now,
$x'(t) = 2t$ and $y'(t) = 2^t ln(2)$.
So,
$x'(2) = 4$ and $y'(t) = 4ln(2)$.
Therefore,
Slope= $\frac{y'(2)}{x'(2)} = \frac{4ln(2)}{4} = ln(2)$
The equation of tangent line is:
$y - 4 = ln(2)(x - 5)$ [y-y$_1 = m(x - x_1)$]