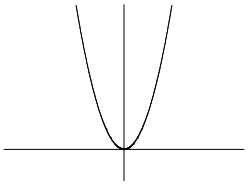
Section 3.5
Maximum and Minimum Values

A quadratic equation is of the form $f(x) = ax^2 + bx + c$, where *a*, *b*, and *c* are real numbers and $a \neq 0$.

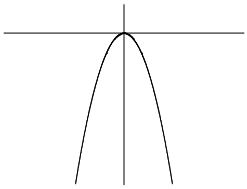
Graphs of Quadratic Functions

The graph of any quadratic function is called a **parabola**.

If $a > 0$ then the parabola will open upward.



If $a < 0$ then the parabola will open downward.



The **vertex** is the turning point of the parabola and is the **minimum point** on the graph when it opens upward and the **maximum point** on the graph when it opens downward. Every parabola has a maximum or minimum, but not both.

The **axis of symmetry** is a vertical line through the vertex that divides the graph in half.

The Standard Form of a Quadratic Function

The quadratic function $f(x) = a(x-h)^2 + k$ is in **standard form**.

The vertex is the point (h, k) and the axis of symmetry is $x = h$.

To be able to put a quadratic equation of the form $f(x) = ax^2 + bx + c$ in the standard form, $f(x) = a(x-h)^2 + k$, you must complete the square on $f(x) = ax^2 + bx + c$.

Example 1: Write each of the following quadratics in standard form. Then find the vertex.

a. $f(x) = x^2 + 2x + 3$

b. $f(x) = x^2 - 12x - 1$

c. $f(x) = -x^2 - 6x$

d. $f(x) = 4x^2 + 8x - 3$

The Vertex f a Parabola Whose Equation is of the Form $f(x) = ax^2 + bx + c$

The parabola's vertex is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a} \right) \right)$.

Recall that the ***maximum or minimum value*** of a quadratic refers to the ***y-value***.

Example 2: Let $f(x) = 2x^2 + 4x + 7$.

a. Does the graph of the function have a minimum or maximum value?

b. Find the min or max value.

Example 3: A company finds that its revenue for selling x units of one of its products is given by the quadratic function $R(x) = -0.5x^2 + 200x$, where R is given in dollars.

a. How many units of this product must be sold so that the maximum revenue is achieved?

b. What is the maximum revenue?