Section 5.1
Exponential Functions
and
Section 5.2
The Number "*e*"

Functions whose equations contain a variable in the exponent are called **exponential functions.**

$f(x) = 2^x$ $\quad$ $g(x) = 7 \cdot 3^{2x-5}$

Real-life situations that can be described using exponential functions:

1. population growth
2. growth of an epidemic
3. radioactive decay
4. other changes that involve rapid increase or decrease

The **exponential function *f* with base *b*** is defined by $f(x) = b^x$ ($b > 0$ and $b \neq 1$) and x is any real number.

If $b = e$ (*the natural base*, $e \approx 2.7183$), then we have $f(x) = e^x$.

- Definition: e is the "limiting value" of $\left(1+\frac{1}{x}\right)^x$ as x grows to infinity.
- It is an irrational number, like π. This means it cannot be written as a fraction nor as a terminating or repeating decimal.

$e \approx 2.7 < 3$

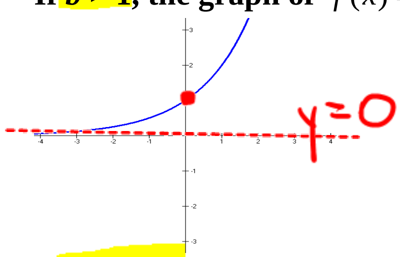
Example 1: Determine whether the following expressions are true or false.

$e^2 < 9$

a. $e^2 > 10$ false

b. $e^{-3} < 27$ true $\quad \frac{1}{e^3} < 1 < 27$

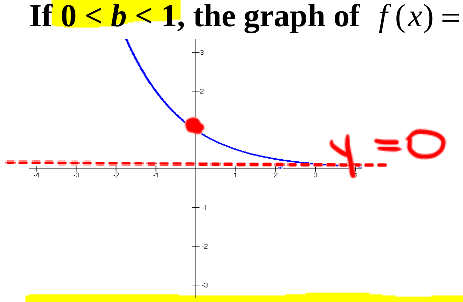
If $b > 1$, the graph of $f(x) = b^x$ looks like (larger b results in a steeper graph):



Domain: $(-\infty, \infty)$

Range: $(0, \infty)$

If $0 < b < 1$, the graph of $f(x) = b^x$ looks like (smaller b results in a steeper graph):



Key points: $(0,1)$

$b^0 = 1,\ b \neq 0$

$(1,b)$

$b^1 = b$

Both graphs have a horizontal asymptote of $y = 0$ (the x-axis).

Transformations of Exponential Functions

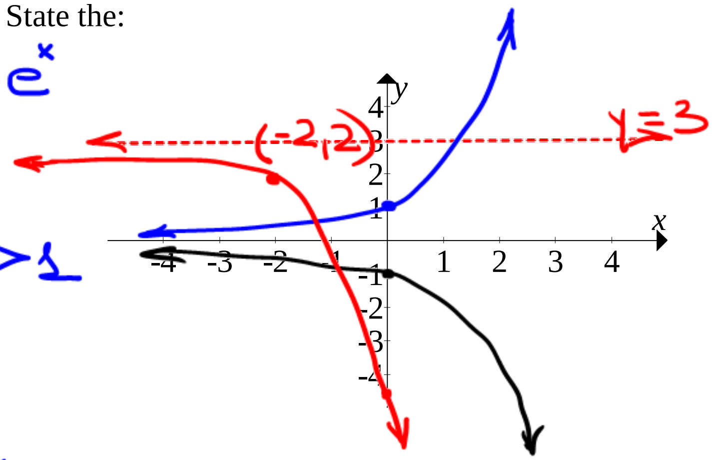
We will apply the same rules we studied in Section 3.4, but since exponential functions have a horizontal asymptote we must remember that when the function has a vertical shift (upward or downward) the horizontal asymptote is shifted by the amount of the vertical shift.

Example 2: Sketch the graph of $f(x) = -e^{x+2} + 3$. State the:

Basic: $y = e^x$

$e > 1$

$e \approx 2.7 > 1$



a. transformations

x-refl, 2 left, 3up

b. domain/range

D: $(-\infty, \infty)$ R: $(-\infty, 3)$

c. asymptote

$y = \overset{+3}{0} \xrightarrow{3\text{up}} \boxed{y = 3}$

d. y-intercept

set $x = 0$ $f(0) = -e^2 + 3 \approx -9 + 3 \approx -6$

e. key point

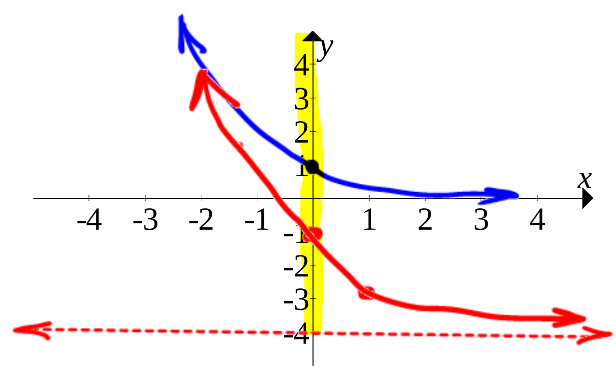
sign

$(0, 1) \xrightarrow{\text{x-refl}} (\overset{-2}{0}, -1) \xrightarrow{2\text{ left}} (-2, \overset{+3}{-1}) \xrightarrow{3\text{ up}} (-2, 2)$

Example 3: Sketch the graph of $f(x) = \left(\frac{1}{3}\right)^{x-1} - 4$. State the:

Basic: $y = \left(\frac{1}{3}\right)^x$

$0 < \frac{1}{3} < 1$



a. transformations

1 right, 4down

b. domain/range

D: $(-\infty, \infty)$ R: $(-4, \infty)$

c. asymptote

$y = \overset{-4}{0} \xrightarrow{4\text{ down}} y = -4$

d. y-intercept

set $x = 0$ $f(0) = \left(\frac{1}{3}\right)^{-1} - 4 = 3 - 4 = -1$

e. key point

$(\overset{+1}{0}, 1) \xrightarrow{1\text{ right}} (1, \overset{-4}{1}) \xrightarrow{4\text{ down}} (1, -3)$

Example 4: Let $f(x) = \left(\frac{1}{3}\right)^{x+1} - 1$

$\text{Basic: } y = \left(\frac{1}{3}\right)^x$

1 left

1 down

a. State its horizontal asymptote.

$y = 0^{-1} \xrightarrow{1 \text{ down}} \boxed{y = -1}$

b. Is the point $\left(0, \frac{-2}{3}\right)$ on the graph of the function?

x f(x)

$-\frac{2}{3} \stackrel{?}{=} \left(\frac{1}{3}\right)^{0+1} - 1$

$-\frac{2}{3} \stackrel{?}{=} \frac{1}{3} - 1$

$-\frac{2}{3} = -\frac{2}{3}$ $\boxed{\text{YES}}$

c. Is the point (-2, -2) on the graph of the function?

x f(x)

$-2 \stackrel{?}{=} \left(\frac{1}{3}\right)^{-2+1} - 1$

$-2 \stackrel{?}{=} \left(\frac{1}{3}\right)^{-1} - 1$

$-2 \stackrel{?}{=} 3 - 1$

$-2 \neq 2$

$\boxed{\text{NO}}$

Example 5: Find the exponential function of the form $f(x) = b^x$ that passes through the points

a. (0, 1) and (4, 81).

x f(x)

$b^4 = 81$

$b = 3$

$\boxed{f(x) = 3^x}$

b. (0, 1) and (3, 64).

x f(x)

$b^3 = 64$

$b = 4$

$\boxed{f(x) = 4^x}$

c) (0, 1) and $\left(3, \frac{1}{8}\right)$

x f(x)

$b^3 = \frac{1}{8}$

$b = \frac{1}{2}$

$\boxed{f(x) = \left(\frac{1}{2}\right)^x}$

$\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{8}$