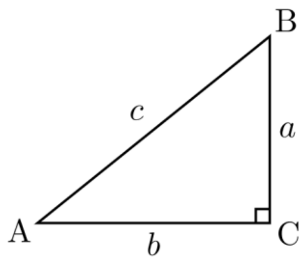
2312 - Section 4.1
Special Triangles and Trigonometric Ratios

First we review some conventions involving triangles and known facts.
It is common to label angles of a triangle with capital letters (Ex.: A, B, and C) and then sides with low case letters a, b, and c with side a opposite angle A, side b opposite angle B, and side c opposite angle C.



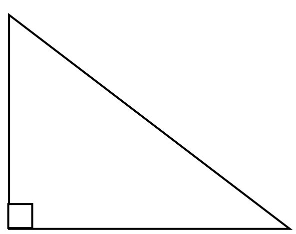
The **sum of measures** of three angles of a triangle is 180°.
An **acute angle** of a triangle is an angle that measures between 0° and 90°.
A **right angle** is a 90° angle.
A right triangle has one right angle. Two other angles are always **acute** and **complementary**, i.e. their measures add up to 90°. The side opposite the right angle is called **hypotenuse**, two other sides are called **legs**.

In this section, we will work with some **special right** triangles before moving on to defining the six trigonometric functions.

If we are given a right triangle and we are interested in finding a missing side, we can apply **Pythagorean's Theorem**

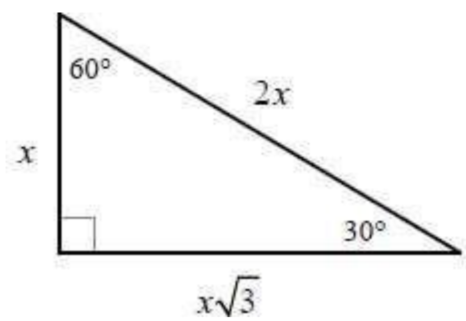
$$a^2 + b^2 = c^2$$

Example 1: In right triangle DOG with the right angle O find OG if $DG = 4\sqrt{5}$ and $DO = 4$.

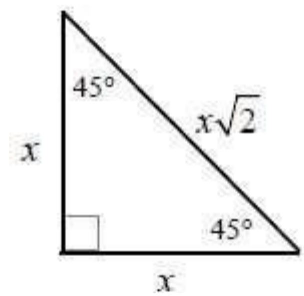


Two **special** triangles are $30^{\circ} - 60^{\circ} - 90^{\circ}$ triangles and $45^{\circ} - 45^{\circ} - 90^{\circ}$ triangles.
In such triangles, sides are proportional. You need to know the length of one side only to find the remaining sides.

$30^{\circ} - 60^{\circ} - 90^{\circ}$ Triangles

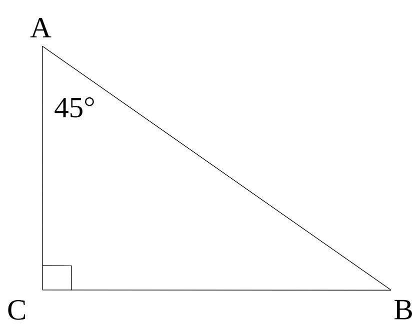


$45^{\circ} - 45^{\circ} - 90^{\circ}$ Triangles

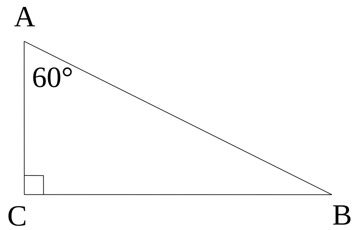


Example 2:

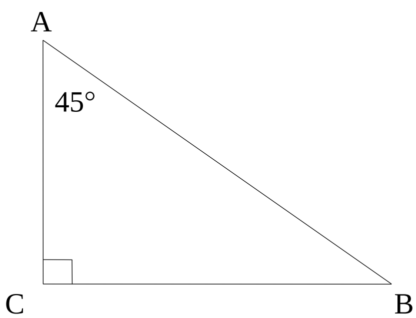
a) *Given:* Right ΔABC with $\angle C = 90^{\circ}$ and $\angle A = 45^{\circ}$; $AC = 7$. *Find*: AB and BC.



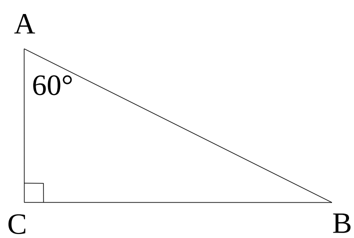
b) *Given:* Right ΔABC with $\angle C = 90^\circ$ and $\angle A = 60^\circ$; $AC = 9$. *Find*: AB and BC.



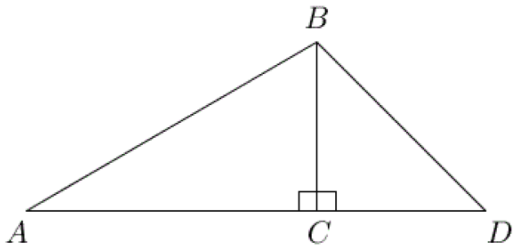
c) *Given:* Right ΔABC with $\angle C = 90^\circ$ and $\angle A = 45^\circ$; $AB = 8$. *Find*: AC and BC.



d) *Given:* Right ΔABC with $\angle C = 90^\circ$ and $\angle A = 60^\circ$; $AB = 20$. *Find*: BC



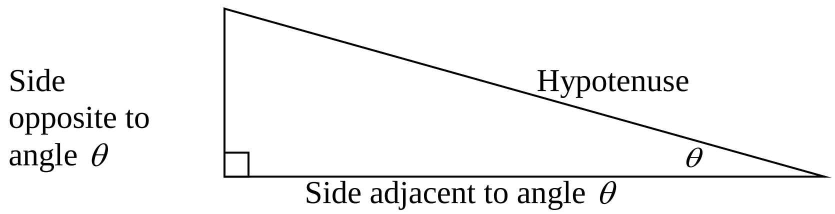
Example 3: Find AC if $\angle A = 30^{\circ}$, $\angle D = 45^{\circ}$, and $BD = 8$.



The Six Trigonometric Functions of an Angle

A trigonometric function is a ratio of the lengths of the sides of a triangle. If we fix an angle, then as to that angle, there are three sides, the **adjacent side**, the **opposite side**, and the **hypotenuse**. We have **six** different combinations of these three sides, so there are six trigonometric functions. The inputs for the trigonometric functions are angles and the outputs are real numbers.

Let θ be an acute angle in a right triangle.



Sine Function:

$$\sin\theta = \frac{length\ of\ the\ side\ opposite\ \theta}{length\ of\ hypotense}$$

Cosine Function:

$$\cos\theta = \frac{length\ of\ the\ side\ adjacent\ to\ \theta}{length\ of\ hypotense}$$

Tangent Function:

$$\tan\theta = \frac{length\ of\ the\ side\ opposite\ \theta}{length\ of\ the\ side\ adjacent\ to\ \theta}$$

Cotangent Function:

$$\cot\theta = \frac{length\ of\ the\ side\ adjacent\ to\ \theta}{length\ of\ the\ side\ opposite\ \theta}$$

Secant Function:

$$\sec\theta = \frac{length\ of\ hypotense}{length\ of\ the\ side\ adjacent\ to\ \theta}$$

Cosecant Function:

$$\csc\theta = \frac{length\ of\ hypotense}{length\ of\ the\ side\ opposite\ \theta}$$

Note: For acute angles, the values of the trigonometric functions are always positive since they are ratios of lengths.

A useful mnemonic device:

SOH-CAH-TOA

$S = \frac{O}{H}$

$C = \frac{A}{H}$

$T = \frac{O}{A}$

$\sin\theta = \frac{opposite}{hypotenuse}$

$\cos\theta = \frac{adjacent}{hypotenuse}$

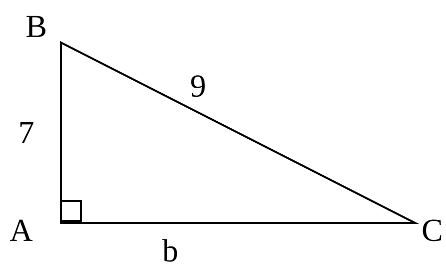
$\tan\theta = \frac{opposite}{adjacent}$

$\cot\theta = \frac{adjacent}{opposite}$

$\sec\theta = \frac{hypotense}{adjacent}$

$\csc\theta = \frac{hypotense}{opposite}$

Example 4: Suppose you are given this triangle. Find the six trigonometric functions of angle B and angle C.



Example 5: Suppose that θ is an acute angle in a right triangle and $\sec\theta = \frac{5\sqrt{3}}{4}$. Find $\sin\theta$ and $\cot\theta$.

Cofunctions
A special relationship exists between sine and cosine, tangent and cotangent, secant and cosecant. The relationship can be summarized as follows:

$$\text{Function of } \theta = \text{Cofunction of the complement of } \theta$$

Recall that two angles are **complementary** if their sum is 90°. In example considered earlier, angles B and C are complementary, since $A = 90^\circ$. Notice that $\sin C = \cos B$ and $B = \cos C$. The same will be true of any pair of cofunctions.

Cofunction relationships

$$\sin A = \cos(90^\circ - A)$$
$$\cos A = \sin(90^\circ - A)$$
$$\tan A = \cot(90^\circ - A)$$
$$\cot A = \tan(90^\circ - A)$$
$$\sec A = \csc(90^\circ - A)$$
$$\csc A = \sec(90^\circ - A)$$

Example 6: Fill in the blanks

a) $\sin 60^\circ = \cos$ ____

b) $\csc 15^\circ =$ ______