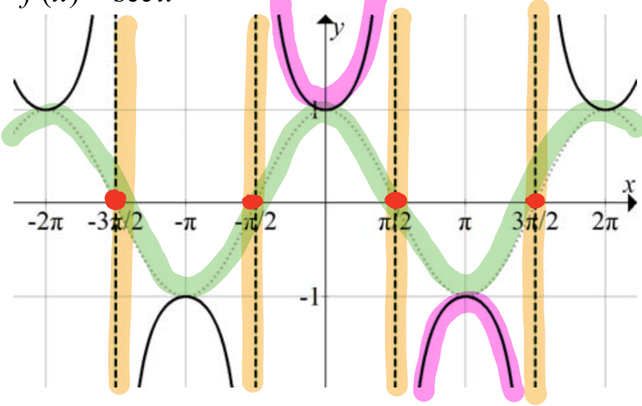
Section 5.3a
Graphs of the Secant and Cosecant Functions

The Secant Graph

RECALL: $\sec x = \dfrac{1}{\cos x}$ so where $\cos x = 0$, $\sec x$ has an asymptote.

To graph $y = A\sec(Bx - C) + D$, first graph, **THE HELPER GRAPH**, $y = A\cos(Bx - C) + D$.

$f(x) = \sec x$



Period: 2π

Vertical Asymptote: $x = \dfrac{k\pi}{2}$, k is an odd integer

Example 1: Let $f(x) = \sec\left(\dfrac{\pi x}{2}\right)$.

a. Give two asymptotes.

$\dfrac{\pi x}{2} = \dfrac{\pi}{2}$ $\quad$ $\dfrac{\pi x}{2} = -\dfrac{\pi}{2}$ $\quad$ $\dfrac{\pi x}{2} = \dfrac{3\pi}{2}$

$\boxed{x = 1}$ $\quad$ $\boxed{x = -1}$ $\quad$ $\boxed{x = 3}$

$$f(x) = \sec\left(\frac{\pi x}{2}\right)$$

b. Sketch its graph by first stating and sketching its helper graph.

Helper function: $y = \cos\left(\frac{\pi x}{2}\right)$ $A=1$ $B=\frac{\pi}{2}$ $C=0$ $D=0$

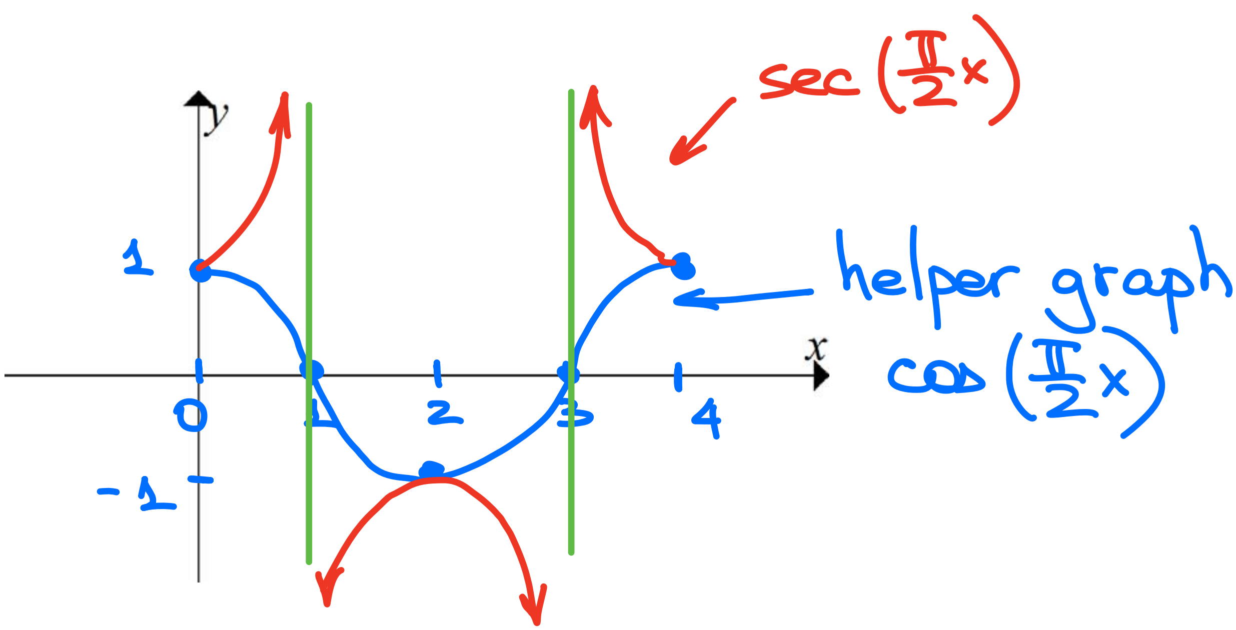
Amplitude: $|A| = 1$

Period: $\frac{2\pi}{B} = \frac{2\pi}{\pi/2} = 2\cancel{\pi} \cdot \frac{2}{\cancel{\pi}} = 4$

Phase Shift: $\frac{C}{B} = \frac{0}{\pi/2} = 0$ $0+4$

One cycle begins at the phase shift and ends at: $\frac{C}{B} + \frac{2\pi}{B}$ $\Longrightarrow [0, 4]$

Any other transformations? no

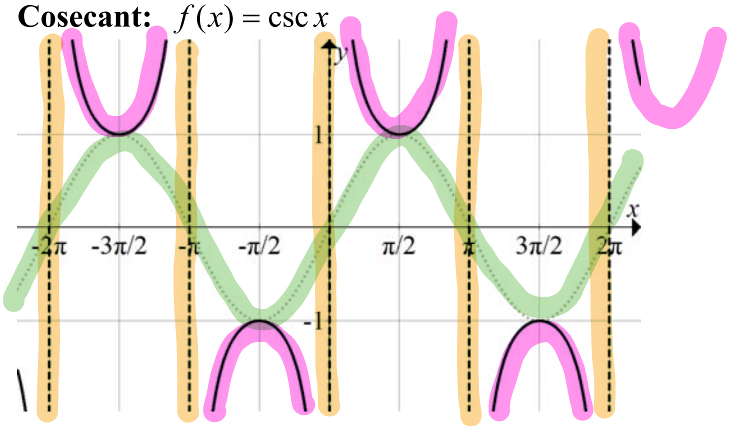


The Cosecant Graph

RECALL: $\csc x = \dfrac{1}{\sin x}$ so where $\sin x = 0$, $\csc x$ has an asymptote.

To graph $y = A\csc(Bx - C) + D$, first graph, **THE HELPER GRAPH**, $y = A\sin(Bx - C) + D$.

Cosecant: $f(x) = \csc x$



Period: 2π

Vertical Asymptote: $x = k\pi$, k is an integer

Example 2: Let $f(x) = 4\csc\left(2x - \dfrac{\pi}{2}\right)$

a. Give two asymptotes.

$2x - \frac{\pi}{2} = 0$

$2x = \frac{\pi}{2}$

$\boxed{x = \frac{\pi}{4}}$

$2x - \frac{\pi}{2} = -\pi$

$2x = -\pi + \frac{\pi}{2}$

$2x = -\frac{\pi}{2}$

$\boxed{x = -\frac{\pi}{4}}$

$f(x) = 4\csc\left(2x - \frac{\pi}{2}\right)$

b. Sketch its graph by first stating and sketching the helper graph.

Helper function: $y = 4\sin\left(2x - \frac{\pi}{2}\right)$ $A = 4$ $B = 2$ $C = \frac{\pi}{2}$ $D = 0$

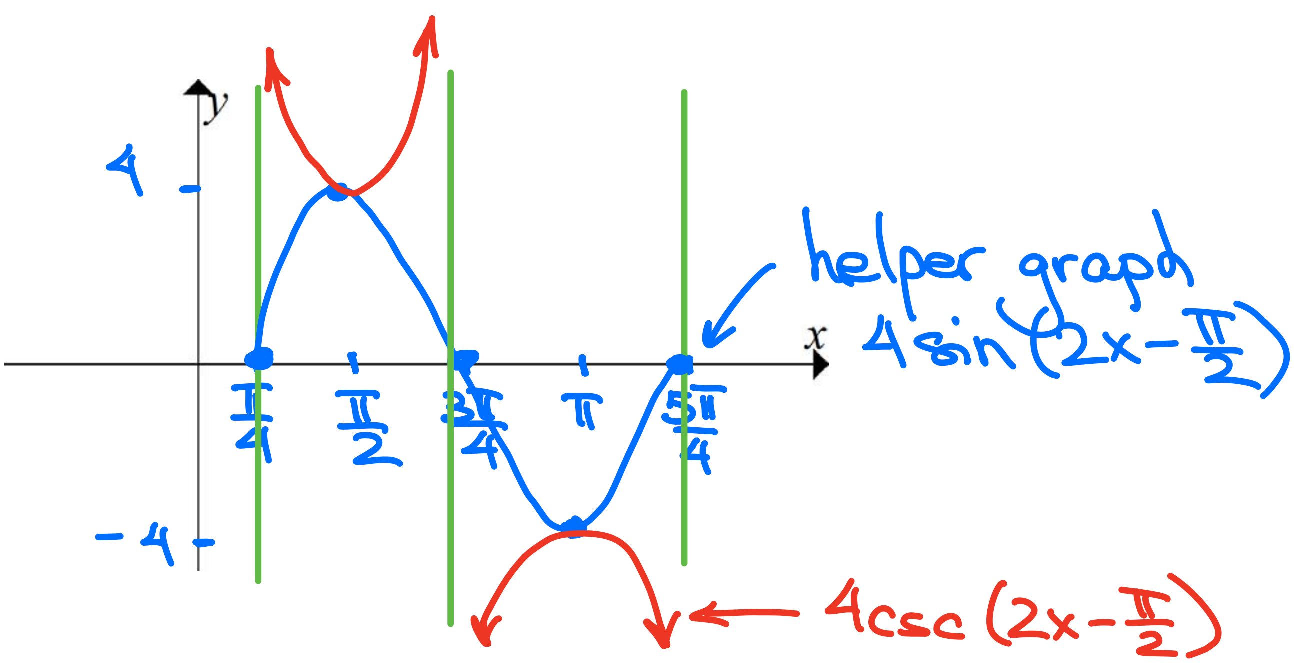
Amplitude: $|A| = 4$

Period: $\frac{2\pi}{B} = \frac{2\pi}{2} = \pi$

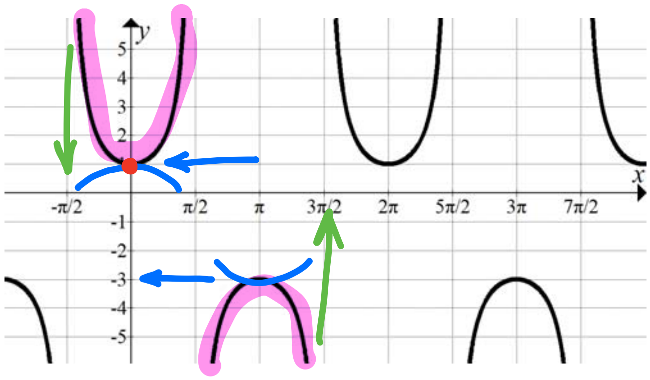
Phase Shift: $\frac{C}{B} = \frac{\pi}{2} \div 2 = \frac{\pi}{2} \cdot \frac{1}{2} = \frac{\pi}{4}$

One cycle begins at the phase shift and ends at: $\frac{C}{B} + \frac{2\pi}{B}$ $\Rightarrow \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

Any other transformations? no

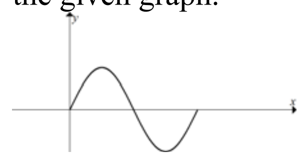


Example 3: Give an equation of the form $f(x) = A\ \csc(Bx - C) + D$ which could be used to represent the given graph. (Note: C or D may be zero.)



Step One:

Let's begin by recalling one cycle of the basic sine graph. Then choose one cycle on the given graph.



Amplitude: $A = \dfrac{M - m}{2} = \dfrac{1-(-3)}{2} = 2$

Vertical Shift, D: It'll be half-way between the maximum and the minimum values.

$D = -1$

Use the period to find B: Recall the period formula $\dfrac{2\pi}{B} = 2\pi \implies B = 1$

$\dfrac{3\pi}{2} - \left(-\dfrac{\pi}{2}\right) = 2\pi$

Compare your chosen cycle to the basic one cycle of sine. Any other transformations?

x-refl

$\frac{\pi}{2}$ right

a. $f(x) = -2\csc\left(x - \frac{1}{2}\pi\right) + 1$

b. $f(x) = -4\csc\left(x - \frac{1}{2}\pi\right) + 1$

c. $f(x) = -2\csc\left(x - \frac{1}{2}\pi\right)$

d. $\boxed{f(x) = -2\csc\left(x - \frac{1}{2}\pi\right) - 1}$

e. $f(x) = -4\csc\left(x - \frac{1}{2}\pi\right) - 1$