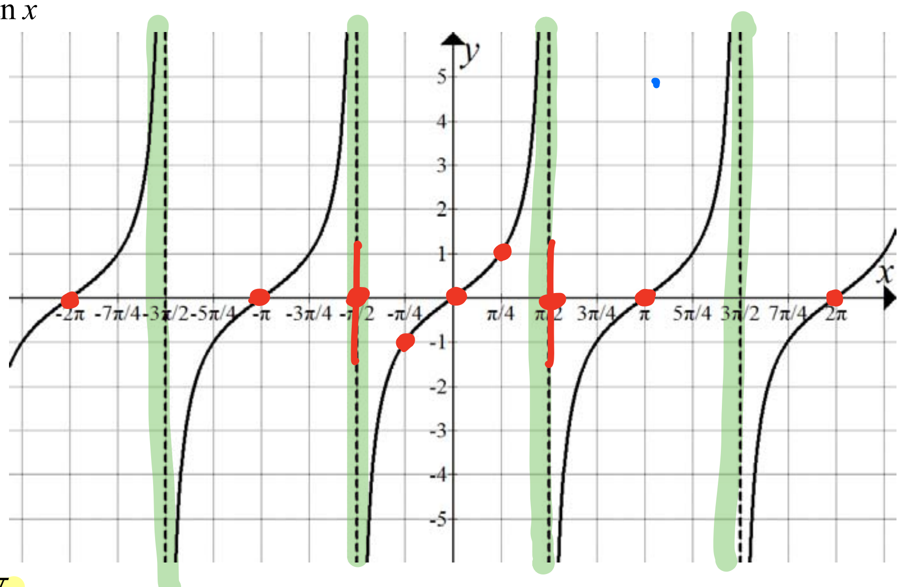
Section 5.3b
Graphs of the Tangent and Cotangent Functions

The Graph of Tangent

Recall: $\tan x = \dfrac{\sin x}{\cos x}$ so where $\cos x = 0$, $\tan x$ has an asymptote and where $\sin x = 0$, $\tan x$ has an x- intercept.

$f(x) = \tan x$



Period: π

Vertical Asymptote: $x = \dfrac{k\pi}{2}$, k is an odd integer.

The period for the function $f(x) = A\tan(Bx - C) + D$ is $\dfrac{\pi}{B}$.

Example 1: Let $f(x) = 5\tan\left(\dfrac{x}{5}\right)$.

a. Find an asymptote.

$\cancel{5} \cdot \dfrac{x}{\cancel{5}} = \dfrac{\pi}{2} \cdot 5$

$\boxed{x = \dfrac{5\pi}{2}}$

b. Find its period.

$B = \dfrac{1}{5}$

$\dfrac{\pi}{1/5} = \pi \cdot \dfrac{5}{1} = \boxed{5\pi}$

How to graph $f(x) = A\tan(Bx - C) + D$**:**

1. Find two consecutive asymptotes by setting Bx – C equal to $-\frac{\pi}{2}$ and $\frac{\pi}{2}$ and then solve for x.
2. Divide the interval connecting the asymptotes found in step 1 into four equal parts. This will create five points. The first and last point is where the asymptotes run through. The middle point is where the x-intercept is located, assuming no vertical shift. The y-coordinates of the second and fourth points are –A and A, assuming no vertical shift.

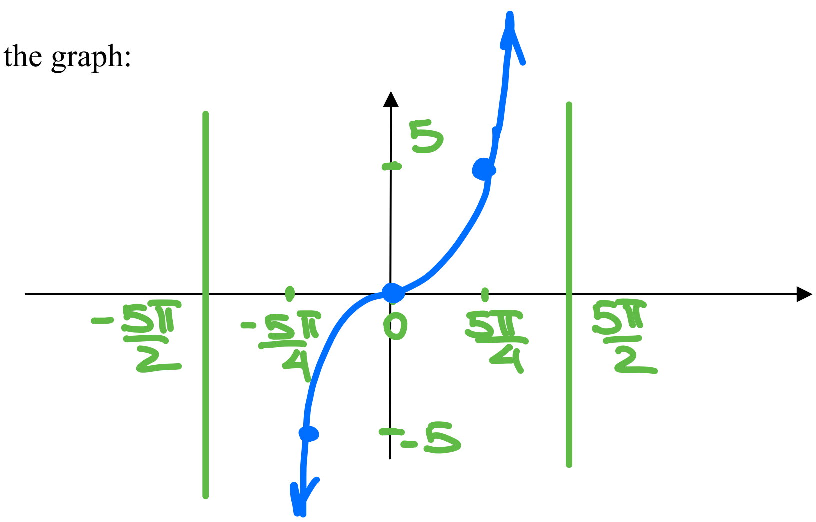
Example 2: Sketch the graph in Example 1: $f(x) = 5\tan\left(\frac{x}{5}\right)$.

$A = 5$

We already found one asymptote, $\frac{5\pi}{2}$ *next find another by setting:* $\cancel{5}\,\frac{x}{\cancel{5}} = -\frac{5\pi}{2}$ *and solve for* x.

$x = -\frac{5\pi}{2}$

Plot theses on the graph:



Now divide the line segment joining the two consecutive asymptotes into four equal parts.

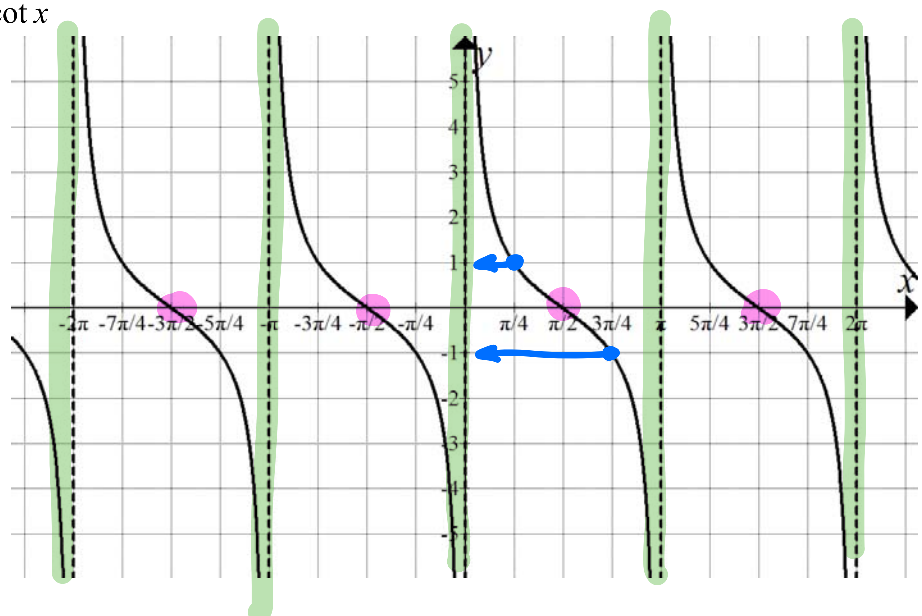
The midpoint will be an x-intercept, assuming no vertical shifting.

The y-coordinates of the second and fourth points are –5 and 5, assuming no vertical shift.

The Graph of Cotangent

Recall: $\cot x = \dfrac{\cos x}{\sin x}$ so where $\cos x = 0$, $\cot x$ has an x- intercept and where $\sin x = 0$, $\cot x$ has an asymptote.

$f(x) = \cot x$



Period: π

Vertical Asymptote: $x = k\pi$, k is an integer.

The period of the function $f(x) = A\cot(Bx - C) + D$ is $\dfrac{\pi}{B}$.

Example 3: Let $f(x) = \cot\left(\pi x - \dfrac{\pi}{3}\right)$.

a. Find an asymptote.

$\pi x - \frac{\pi}{3} = 0$

$\pi x = \frac{\pi}{3}$

$\boxed{x = \frac{1}{3}}$

b. Find its period.

$B = \pi$

$\frac{\pi}{B} = \frac{\pi}{\pi} = \boxed{1}$

How to graph $f(x) = A\cot(Bx - C) + D$**:**

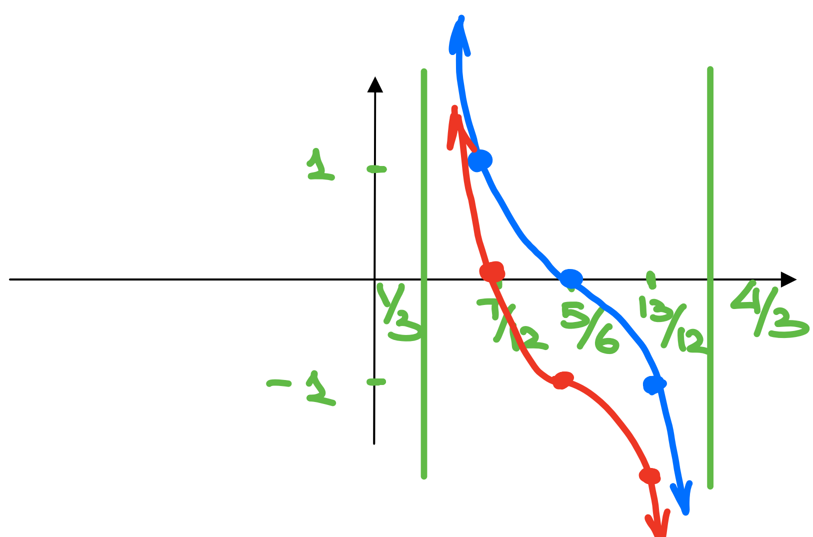
1. Find two consecutive asymptotes by setting Bx – C equal to 0 and π and then solve for x.
2. Divide the interval connecting the asymptotes found in step 1 into four equal parts. This will create five points. The first and last point is where the asymptotes run through. The middle point is where the x-intercept is located, assuming no vertical shift. The y-coordinates of the second and fourth points are –A and A, assuming no vertical shift.

down 1

Example 4: Sketch the graph in Example 3: $f(x) = \cot\left(\pi x - \frac{\pi}{3}\right) - 1$.

We already found one asymptote, $\frac{1}{3}$ *next find another by setting:* $\pi x - \frac{\pi}{3} = \pi$ *and solve for x.*

$\pi x = \frac{4\pi}{3}$ $x = \frac{4}{3}$



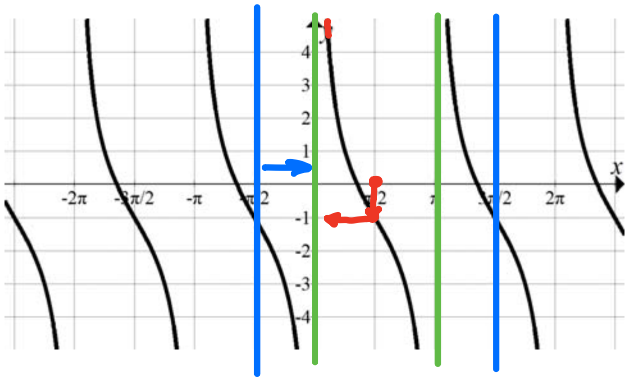
Now divide the line segment joining the two consecutive asymptotes into four equal parts.

The midpoint will be an x-intercept, assuming no vertical shifting.

The y-coordinates of the second and fourth points are –1 and 1, assuming no vertical shift.

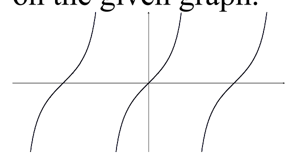
Lastly, shift vertically 1 unit.

Example 5: Give an equation of the form $f(x) = A\tan(Bx - C) + D$ which could be used to represent the given graph. (Note: C or D may be zero.)



Step One:

Let's begin by recalling one cycle of the basic tangent graph. Then choose one cycle on the given graph.



Based on the cycle from the graph you chose, note the consecutive asymptotes.

Vertical Shift, D:
Find the midpoint. Is it on the x-axis? If so, there is no vertical shift, or D = 0.
Otherwise, D = -1

Use the period to fi

nd B: Recall the period formula $\frac{\pi}{B} = \pi \Rightarrow B = 1$

Compare your chosen cycle with the basic one cycle of tangent. Any other transformations?

a. $f(x) = -2\tan\left(x - \frac{1}{2}\pi\right) + 1$

b. $f(x) = -2\tan(x)$

c. $f(x) = -2\tan\left(x - \frac{1}{2}\pi\right)$

d. $f(x) = -2\tan(x+1) - 1$

e. $f(x) = -2\tan\left(x - \frac{1}{2}\pi\right) - 1$